18 Coordinate Geometry

INTRODUCTION

Coordinate geometry is that branch of Mathematics which deals with the study of geometry by means of algebra. René Déscartes, a French mathematician, realised around 1637 that a line or a curve in a plane can be represented by an algebraic equation. As a result, a new branch of mathematics called **Coordinate Geometry** came into existence. In coordinate geometry, we represent a point in a plane by an ordered pair of real numbers, called coordinates of the point; and a straight line or a curve by an algebraic equation with real coefficients. Thus, we use algebra advantageously to the study of straight lines and geometric curves.

Recall that there is one and only one point on a number line associated with each real number. A similar situation exists for points in a plane and **ordered pairs** of real numbers.

18.1 ORDERED PAIR

*An **ordered pair** is a pair of objects taken in a specific order.*

An ordered pair is written by listing its two members in a specific order, separating them by a comma and enclosing the pair in parentheses. In the ordered pair (a, b), a is called the *first member* (or *component*) and b is called the *second member* (or *component*).

Equality of ordered pairs. *Two ordered pairs (a, b) and (c, d) are called **equal**, written as $(a, b) = (c, d)$, if and only if $a = c$ and $b = d$.*

Remarks

- The word 'ordered' implies that the order in which the two elements of the pair occur is meaningful. For example, if we have a sock and a shoe, the order in which they are put on does matter.
- The ordered pairs (a, b) and (b, a) are different unless $a = b$.
- The two components of an ordered pair may be equal.

18.2 COORDINATE SYSTEM

When two numbered lines perpendicular to each other (usually horizontal and vertical) are placed together so that the two origins (the points corresponding to zero) coincide then the resulting configuration is called a **cartesian coordinate system** or simply a **coordinate system** or a **cartesian plane**.

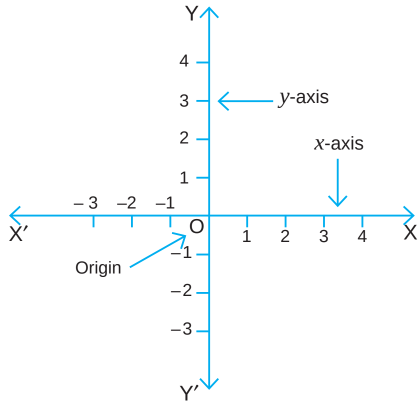
Let X′OX and Y′OY, two number lines perpendicular to each other, meet at the point O (shown in the adjoining figure), then

(*i*) X′OX is called ***x*-axis.**

(*ii*) Y′OY is called ***y*-axis.**

(*iii*) X′OX and Y′OY taken together are called **coordinate axes.**

(*iv*) the point O is called the **origin**.



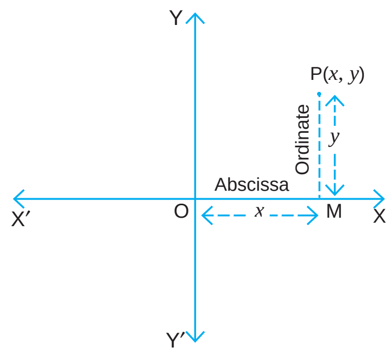
18.2.1 Coordinates of a point

Let P be any point in the coordinate plane. From P, draw PM perpendicular to X′OX, then

(*i*) OM is called ***x*-coordinate** or **abscissa** of P and is usually denoted by x.

(*ii*) MP is called ***y*-coordinate** or **ordinate** of P and is usually denoted by y.

(*iii*) x and y taken together are called **cartesian coordinates** or simply coordinates of P and are denoted by (x, y).



Remarks

- To know the position of a point in a plane, we need two independent informations abscissa and ordinate of the point.
- The coordinates of a point indicate its position with reference to coordinate axes.
- In stating the coordinates of a point, the ***abscissa*** precedes the ***ordinate***. The two are separated by a comma and are enclosed in the bracket ( ). Thus, a point P whose abscissa is 'x' and ordinate is 'y' is written as (x, y) or P (x, y).

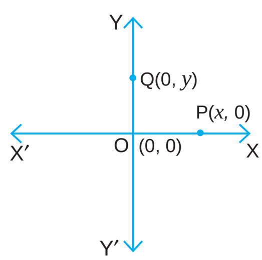
❒ Convention for signs of coordinates

(*i*) The x-coordinate (abscissa) of a point is ***positive*** if it is measured to the right of O *i.e.* along OX and is ***negative*** if it is measured to the left of O *i.e.* along OX′.

(*ii*) The y-coordinate (ordinate) of a point is ***positive*** if it is measured upwards *i.e.* along OY and is ***negative*** if it is measured downwards *i.e.* along OY′.

Remarks

- The coordinates of the origin O are (0, 0).
- For any point on x-axis, its ordinate is always zero and so the coordinates of any point P on x-axis are $(x, 0)$.
- For any point on y-axis, its abscissa is always zero and so the coordinates of any point Q on y-axis are $(0, y)$.



Quadrants

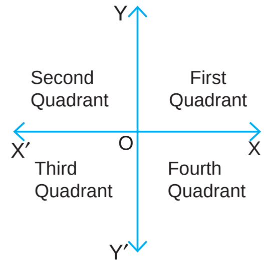
The horizontal and the vertical number lines X′OX and Y′OY divide the coordinate plane into four parts called **quadrants**.

(*i*) XOY is called the **first quadrant**. In this quadrant, $x > 0$, $y > 0$ *i.e.* abscissa and ordinate are both positive.

(*ii*) YOX′ is called the **second quadrant**. In this quadrant, $x < 0$, $y > 0$ *i.e.* abscissa is negative and ordinate is positive.

(*iii*) X′OY′ is called the **third quadrant**. In this quadrant, $x < 0$, $y < 0$ *i.e.* abscissa and ordinate are both negative.

(*iv*) Y′OX is called the **fourth quadrant**. In this quadrant, $x > 0$, $y < 0$ *i.e.* abscissa is positive and ordinate is negative.



The signs of the coordinates of a point determine the quadrant in which the point lies and conversely, the signs of the coordinates of a point are determined by the quadrant in which it lies. The signs of the coordinates of a point in four quadrants can be remembered with the help of the following table:

Quadrants → *Coordinates* ↓	*1st XOY*	*2nd YOX′*	*3rd X′OY′*	*4th Y′OX*
x (abscissa)	+ve	−ve	−ve	+ve
y (ordinate)	+ve	+ve	−ve	−ve

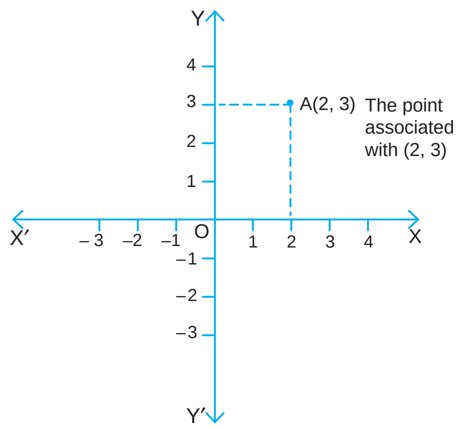
Thus, the points (2, 3), (–2, 3), (–2, –3) and (2, –3) lie in the first, second, third and fourth quadrants respectively.

18.2.2 Plotting of points

When we identify a point in the coordinate plane with a given ordered pair of real numbers, we say that we **plot** the point. For example, to plot the point (2, 3), we adopt two steps:

(*i*) Start from O (origin) and move 2 units along the x-axis to the right.

(*ii*) From this place, move 3 units upwards (parallel to y-axis) and mark a ***dot*** at that place. This point, say A, of the coordinate plane is the point associated with the given ordered pair (2, 3). Label this point as A (2, 3).

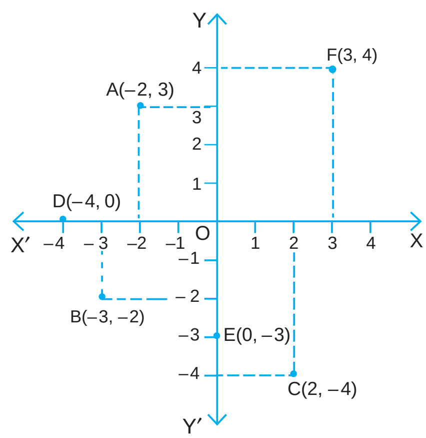


Illustrative Examples

Example 1. *Plot the points associated with the pairs A*(–2, 3), *B*(–3, –2), *C* (2, –4), *D* (–4, 0), *E*(0, –3) *and F*(3, 4).

Solution. To plot the point A (–2, 3) in the coordinate plane, start from the point O (origin) and move 2 units along the x-axis to the left, and from here move 3 units upwards. Mark a ***dot*** at this place, and thus the point A (–2, 3) is plotted.

Similarly, plot the other points in the plane. These points are shown in the adjacent figure.

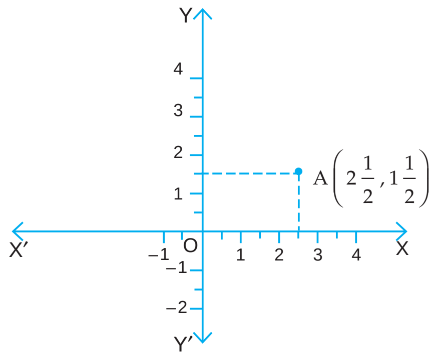


Example 2. *Plot the following points on a squared paper:*

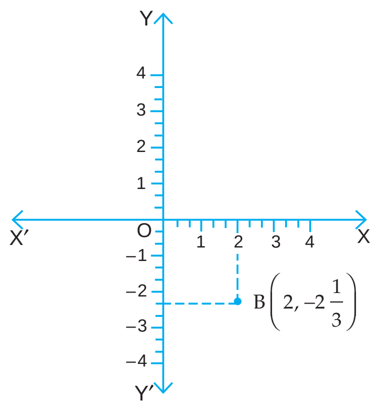
(i) $A\left(2\frac{1}{2}, 1\frac{1}{2}\right)$ *(ii)* $B\left(2, -2\frac{1}{3}\right)$ *(iii)* $\left(1\frac{1}{2}, 1\frac{1}{3}\right)$.

Solution.

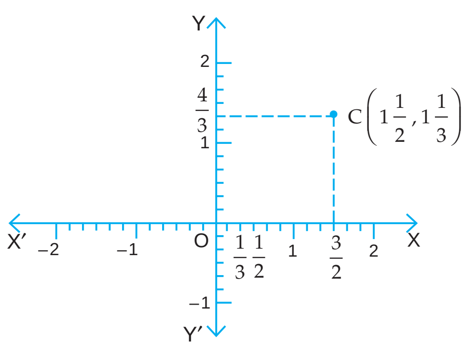
(*i*) To plot the point A $\left(2\frac{1}{2}, 1\frac{1}{2}\right)$ on a squared (graph) paper, mark the two coordinate axes in such a way that the fraction $\frac{1}{2}$ can easily be read. For this, take 2 divisions equal to one unit. The point A $\left(2\frac{1}{2}, 1\frac{1}{2}\right)$ is shown by a dot in the adjacent figure.



(*ii*) To plot the point B $\left(2, -2\frac{1}{3}\right)$ on a squared (graph) paper, mark the two coordinate axes in such a way that the fraction $\frac{1}{3}$ can easily be read. For this, take 3 divisions equal to one unit. The point B $\left(2, -2\frac{1}{3}\right)$ is shown by a dot in the adjacent figure.

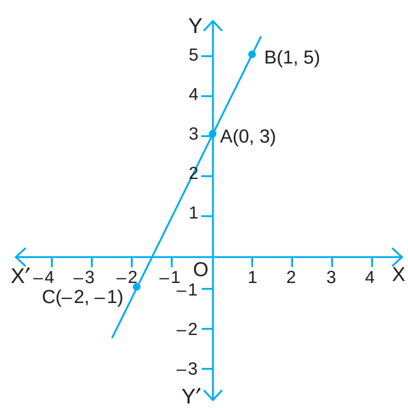


(*iii*) To plot the point C $\left(1\frac{1}{2}, 1\frac{1}{3}\right)$ on a squared paper, mark the coordinate axes in such a way that the fractions $\frac{1}{2}$ and $\frac{1}{3}$ both can easily be read. For this, take 6 divisions equal to one unit. The point C $\left(1\frac{1}{2}, 1\frac{1}{3}\right)$ is shown by a dot in the adjacent figure.

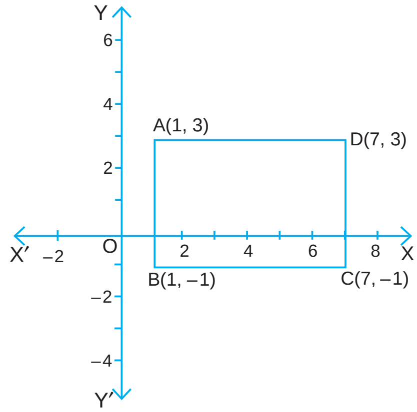


Example 3. *Plot the points A (0, 3), B (1, 5) and C (–2, –1) on a graph paper and check whether they are collinear (lie on the same straight line) or not.*

Solution. Plot the points A(0, 3), B(1, 5) and C (–2, –1) on the graph paper as usual. On joining the points A and B by a straight line, we find that the point C(–2, –1) lies on this line. Hence, the given points A(0, 3), B (1, 5) and C (–2, –1) are collinear.



Example 4. *Three vertices (corners) of a rectangle are A(1, 3), B(1, –1) and C(7, –1). Plot these points on a graph paper and hence, use it to find the coordinates of the fourth vertex. Also find the area of the rectangle.*



Solution. Plot the points A (1, 3), B (1, –1) and C (7, –1) on the graph paper as usual. Join the points to complete the rectangle ABCD as shown in the adjacent figure. Now read the coordinates of the point D from the graph paper. Clearly, the point D is (7, 3), and length of rectangle = 6 units and breadth = 4 units.

∴ Area of rectangle ABCD = (6 × 4) sq. units

= 24 sq. units.

Example 5. *Three vertices of a parallelogram are A(–2, 2), B(6, 2) and C(4, –3). Plot these points on a graph paper and hence, use it to find the coordinates of the fourth vertex D. Also find the coordinates of the mid-point of the side CD. What is the area of the parallelogram?*

Solution. Plot the points A(–2, 2), B(6, 2) and C(4, –3) as usual.

Complete the parallelogram ABCD.

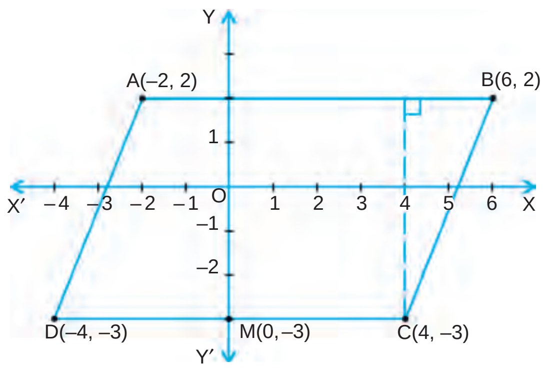
From graph, the coordinates of the point D are (–4, –3).

The coordinates of the mid-point M of CD are (0, –3).

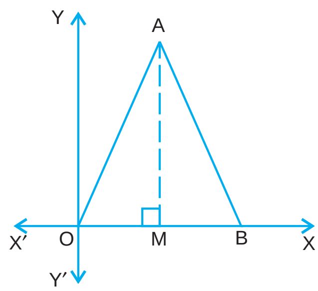
From figure, CD = 8 units and the height of the || gm ABCD corresponding to the side CD = 5 units.

∴ The area of || gm ABCD

= (8 × 5) sq. units = 40 sq. units.



Example 6. *The adjoining figure shows an isosceles triangle OAB with sides OA = AB = 13 units and OB = 10 units. Find the coordinates of the vertices.*



Solution. From A, draw AM ⊥ OB.

Then M is mid-point of OB,

so OM = MB = 5 units.

In ΔAOM, ∠M = 90°. By Pythagoras theorem, we get

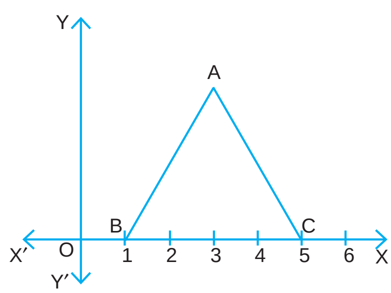
$OA^2 = OM^2 + AM^2 \Rightarrow 13^2 = 5^2 + AM^2$

$\Rightarrow$ $AM^2 = 169 - 25 = 144$

$\Rightarrow$ AM = 12 units.

Clearly, coordinates of O and B are (0, 0) and (10, 0) respectively. As OM = 5 units and AM = 12 units, therefore, coordinates of A are (5, 12).

Example 7. *In the adjoining figure, ABC is an equilateral triangle. Find the coordinates of the vertices.*



Solution. From figure, BC = 4 units.

As ABC is an equilateral triangle,

AB = AC = 4 units.

From A, draw AM ⊥ BC, then M is mid-point of BC.

So, BM = $\frac{1}{2}$ BC = $\frac{1}{2}$ × 4 units = 2 units.

In ΔABM, $\angle M = 90°$. By Pythagoras theorem, we get

$$AB^2 = AM^2 + BM^2$$

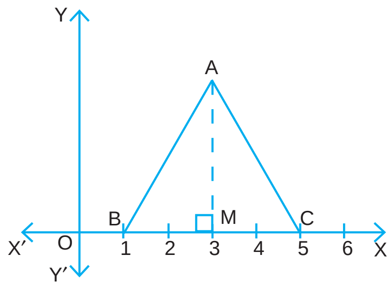
$\Rightarrow \quad 4^2 = AM^2 + 2^2$

$\Rightarrow \quad AM^2 = 16 - 4 = 12 \Rightarrow AM = 2\sqrt{3}$ units.

From figure, OM = OB + BM = (1 + 2) units = 3 units.

Clearly, coordinates of B and C are (1, 0) and (5, 0) respectively.

As OM = 3 units and AM = $2\sqrt{3}$ units, therefore, coordinates of A are $(3, 2\sqrt{3})$.



Exercise 18.1

1. Find the coordinates of points whose

(*i*) abscissa is 3 and ordinate –4.

(*ii*) abscissa is $-\frac{3}{2}$ and ordinate 5.

(*iii*) whose abscissa is $-1\frac{2}{3}$ and ordinate $-2\frac{1}{4}$.

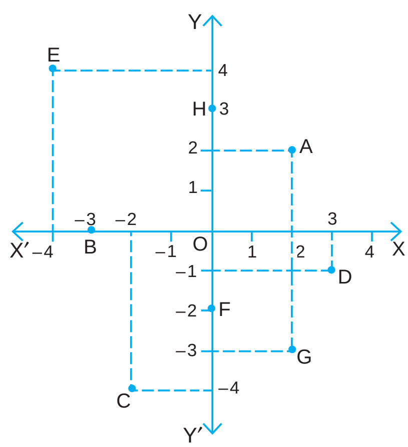
(*iv*) whose ordinate is 5 and abscissa is –2.

(*v*) whose abscissa is –2 and lies on *x*-axis.

(*vi*) whose ordinate is $\frac{3}{2}$ and lies on *y*-axis.

2. Plot the following points on the same graph paper :
A(3, 4), B(–3, 1), C(1, –2), D(–2, –3), E(0, 5), F(5, 0), G(0, –3), H(–3, 0).

3. Write the coordinates of the points A, B, C, D, E, F, G and H shown in the adjacent figure.



4. In which quadrants are the points A, B, C and D of problem 3 located?

5. Plot the following points on the same graph paper:

$A\left(2, \frac{5}{2}\right), B\left(-\frac{3}{2}, 3\right), C\left(\frac{1}{2}, -\frac{3}{2}\right)$ and $D\left(-\frac{5}{2}, -\frac{1}{2}\right)$.

6. Plot the following points on the same graph paper:

$A\left(\frac{4}{3}, -1\right), B\left(\frac{7}{2}, \frac{5}{3}\right), C\left(\frac{13}{6}, 0\right), D\left(-\frac{5}{3}, -\frac{5}{2}\right)$.

7. Plot the following points and check whether they are collinear or not:

(*i*) (1, 3), (–1, –1) and (–2, –3)

(*ii*) (1, 2), (2, –1) and (–1, 4)

(*iii*) (0, 1), (2, –2) and $\left(\frac{2}{3}, 0\right)$.