INTRODUCTION
If we put the sharp tip of a pencil on a sheet of paper and move from one point to the other, without lifting the pencil, then the shapes so formed are called plane curves.

Some plane curves are shown below:

(i) \hspace{1cm} (ii) \hspace{1cm} (iii) \hspace{1cm} (iv)

(v) \hspace{1cm} (vi) \hspace{1cm} (vii)

The (plane) curves which have different beginning and end points are called open curves and the curves which have same beginning and end points are called closed curves. In the above figure, (i), (iv) and (vi) are open curves whereas (ii), (iii), (v) and (vii) are closed curves.

A curve which does not cross itself at any point is called a simple curve. In the above figure, (i), (ii), (iv), (v) and (vii) are all simple curves. Note that (ii), (v) and (vii) are simple closed curves. A simple closed plane curve made up entirely of line segments is called a polygon. In the above figure, (v) and (vii) are polygons.

In this chapter, we shall study about different kinds of polygons (parallelograms, rectangles, rhombuses, squares, kites) and their various properties. We shall also construct some quadrilaterals and regular hexagons by using ruler and compass.
12.1 RECTILINEAR FIGURES

A plane figure made up entirely of line segments is called a rectilinear figure. Look at the following plane figures:

- (i)
- (ii)
- (iii)
- (iv)
- (v)
- (vi)

All the six figures are made up entirely of line segments, so these are all rectilinear figures.

Polygon

A polygon is a simple closed rectilinear figure i.e. a polygon is a simple closed plane figure made up entirely of line segments.

The figures (shown above) (i), (ii), (iv) and (vi) are polygons where as figures (iii) and (v) are not polygons since figure (iii) is not simple and figure (v) is not closed.

Thus, every polygon is a rectilinear figure but every rectilinear figure is not a polygon.

The line segments are called its sides and the points of intersection of consecutive sides are called its vertices. An angle formed by two consecutive sides of a polygon is called an interior angle or simply an angle of the polygon.

A polygon is named according to the number of sides it has.

<table>
<thead>
<tr>
<th>No. of sides</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name</td>
<td>Triangle</td>
<td>Quadrilateral</td>
<td>Pentagon</td>
<td>Hexagon</td>
<td>Heptagon</td>
<td>Octagon</td>
<td>Decagon</td>
</tr>
</tbody>
</table>

Diagonal of a polygon. Line segment joining any two non-consecutive vertices of a polygon is called its diagonal.

Convex polygon. If all the (interior) angles of a polygon are less than 180°, it is called a convex polygon.

In the adjoining figure, ABCDEF is a convex polygon. In fact, it is a convex hexagon.

Concave polygon. If one or more of the (interior) angles of a polygon is greater than 180° i.e. reflex, it is called concave (or re-entrant) polygon.

In the adjoining figure, ABCDEFG is a concave polygon. In fact, it is a concave pentagon.

However, we shall be dealing with convex polygons only.
Exterior angle of convex polygon

If we produce a side of a polygon, the angle it makes with the next side is called an exterior angle.

In the adjoining figure, ABCDE is a pentagon. Its side AB has been produced to P, then \( \angle \text{CBP} \) is an exterior angle.

Note that corresponding to each interior angle, there is an exterior angle. Also, as an exterior angle and its adjacent interior angle make a line, so we have:

\[ \text{an exterior angle} + \text{adjacent interior angle} = 180^\circ. \]

Regular polygon. A polygon is called regular polygon if all its sides have equal length and all its angles have equal size.

Thus, in a regular polygon:

(i) all sides are equal in length
(ii) all interior angles are equal in size
(iii) all exterior angles are equal in size.

All regular polygons are convex. All equilateral triangles and all squares are regular polygons.

12.2 Quadrilaterals

A simple closed plane figure bounded by four line segments is called a quadrilateral.

In the adjoining figure, ABCD is a quadrilateral.

It has

four sides — AB, BC, CD and DA
four (interior) angles — \( \angle A, \angle B, \angle C \) and \( \angle D \)
four vertices — A, B, C and D
two diagonals — AC and BD.

In quadrilateral ABCD, sides AB, BC; BC, CD; CD, DA; DA, AB are pairs of adjacent sides.

Sides AB, CD; BC, DA are pairs of opposite sides.

Angles \( \angle A, \angle B; \angle B, \angle C; \angle C, \angle D; \angle D, \angle A \) are pairs of adjacent angles.

Angles \( \angle A, \angle C; \angle B, \angle D \) are pairs of opposite angles.

Angle sum property of a quadrilateral

Sum of (interior) angles of a quadrilateral is 360°.

In an adjoining figure, ABCD is any quadrilateral. Diagonal AC divides it into two triangles. We know that the sum of angles of a triangle is 180°,

in \( \triangle ABC \), \( \angle 1 + \angle B + \angle 2 = 180^\circ \) …(i)

in \( \triangle ACD \), \( \angle 4 + \angle D + \angle 3 = 180^\circ \) …(ii)

On adding (i) and (ii), we get

\[ \angle 1 + \angle 4 + \angle B + \angle D + \angle 2 + \angle 3 = 360^\circ \]

\[ \angle A + \angle B + \angle D + \angle C = 360^\circ \] (from figure)

Hence, the sum of (interior) angles of a quadrilateral is 360°.
Types of quadrilaterals

1. Trapezium

A quadrilateral in which one pair of opposite sides is parallel is called a trapezium (abbreviated trap.).

The parallel sides are called bases of the trapezium. The line segment joining mid-points of non-parallel sides is called its median.

In the adjoining quadrilateral, AB \parallel DC whereas AD and BC are non-parallel, so ABCD is a trapezium, AB and CD are its bases, and EF is its median where E, F are mid-points of the sides AD, BC respectively.

Isosceles trapezium

If non-parallel sides of a trapezium are equal, then it is called an isosceles trapezium.

Here AB \parallel DC, AD and BC are non-parallel and AD = BC, so ABCD is an isosceles trapezium.

2. Parallelogram

A quadrilateral in which both pairs of opposite sides are parallel is called a parallelogram. It is usually written as ‘\parallel gm’.

In the adjoining quadrilateral, AB \parallel DC and AD \parallel BC, so ABCD is a parallelogram.

3. Rectangle

If one of the angles of a parallelogram is a right angle, then it is called a rectangle.

In the adjoining parallelogram, \angle A = 90^\circ, so ABCD is a rectangle. Of course, the remaining angles will also be right angles.

4. Rhombus

If all the sides of a quadrilateral are equal, then it is called a rhombus.

In the adjoining quadrilateral, AB = BC = CD = DA, so ABCD is a rhombus.

[Every rhombus is a parallelogram, see corollary to theorem 12.3]

5. Square

If two adjacent sides of a rectangle are equal, then it is called a square. Alternatively, if one angle of a rhombus is a right angle, it is called a square.

In the adjoining rectangle, AB = AD, so ABCD is a square. Of course, the remaining sides are also equal.
6. Kite

A quadrilateral in which two pairs of adjacent sides are equal is called a kite (or diamond).

In the adjoining quadrilateral, AD = AB and DC = BC, so ABCD is a kite.

**Remark**

From the above definitions it follows that parallelograms include rectangles, squares and rhombi (plural of rhombus), therefore, any result which is true for a parallelogram is certainly true for all these figures.

### 12.2.1 Properties of parallelograms

#### Theorem 12.1

A diagonal of a parallelogram divides it into two congruent triangles.

**Given.** A parallelogram ABCD and diagonal AC divides it into two triangles ΔABC and ΔCDA.

**To prove.** ΔABC ≅ ΔCDA.

**Proof.**

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>In ΔABC and ΔCDA</td>
<td></td>
</tr>
<tr>
<td>1. ∠BAC = ∠ACD</td>
<td>1. Alt. ∠s, AB</td>
</tr>
<tr>
<td>2. ∠BCA = ∠CAD</td>
<td>2. Alt. ∠s, BC</td>
</tr>
<tr>
<td>3. AC = CA</td>
<td>3. Common</td>
</tr>
<tr>
<td>4. ΔABC ≅ ΔCDA</td>
<td>4. ASA rule of congruency.</td>
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</tbody>
</table>

#### Theorem 12.2

In a parallelogram, opposite sides are equal.

**Given.** A parallelogram ABCD.

**To prove.** AB = DC and BC = AD.

**Construction.** Join AC.

**Proof.**

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<td>4. ASA rule of congruency.</td>
</tr>
<tr>
<td>5. AB = DC and BC = AD</td>
<td>5. c.p.c.t.</td>
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</table>
The converse of the above theorem is also true. In fact we have:

**Theorem 12.3**

*If each pair of opposite sides of a quadrilateral is equal, then it is a parallelogram.*

*Given.* A quadrilateral ABCD in which AB = DC and BC = AD.

*To prove.* ABCD is a parallelogram.

*Construction.* Join AC.

*Proof.*

<table>
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<tr>
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<tbody>
<tr>
<td>In ( \triangle ABC ) and ( \triangle CDA )</td>
<td></td>
</tr>
<tr>
<td>1. ( AB = DC )</td>
<td>1. Given.</td>
</tr>
<tr>
<td>2. ( BC = AD )</td>
<td>2. Given.</td>
</tr>
<tr>
<td>3. ( AC = AC )</td>
<td>3. Common.</td>
</tr>
<tr>
<td>4. ( \triangle ABC \cong \triangle CDA )</td>
<td>4. SSS rule of congruency</td>
</tr>
<tr>
<td>5. ( \angle BAC = \angle ACD ) ( \Rightarrow ) ( AB \parallel DC )</td>
<td>5. c.p.c.t. alt. ( \angle s ) are equal formed by lines ( AB, DC ) and transversal ( AC ).</td>
</tr>
<tr>
<td>6. ( \angle ACB = \angle CAD ) ( \Rightarrow ) ( BC \parallel AD )</td>
<td>6. c.p.c.t. alt. ( \angle s ) are equal formed by lines ( AD, BC ) and transversal ( AC ).</td>
</tr>
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</table>

Hence, ABCD is a parallelogram.

**Corollary.** Every rhombus is a parallelogram.

[In a rhombus, all sides are equal, so opposite sides are equal. Therefore, every rhombus is a parallelogram.]

**Theorem 12.4**

*In a parallelogram, opposite angles are equal.*

*Given.* A parallelogram ABCD.

*To prove.* \( \angle A = \angle C \) and \( \angle B = \angle D \).

*Proof.*

<table>
<thead>
<tr>
<th>Statements</th>
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</tr>
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<tbody>
<tr>
<td>1. ( \angle A + \angle B = 180^\circ )</td>
<td>1. ( AD \parallel BC ) and ( AB ) is a transversal, sum of co-interior angles = ( 180^\circ )</td>
</tr>
<tr>
<td>2. ( \angle B + \angle C = 180^\circ )</td>
<td>2. ( AB \parallel DC ) and ( BC ) is a transversal, sum of co-interior angles = ( 180^\circ )</td>
</tr>
<tr>
<td>3. ( \angle A + \angle B = \angle B + \angle C ) ( \Rightarrow ) ( \angle A = \angle C ) Similarily, ( \angle B = \angle D ).</td>
<td>3. From 1 and 2</td>
</tr>
</tbody>
</table>

The converse of the above theorem is also true. In fact, we have:

**Theorem 12.5**

*If each pair of opposite angles of a quadrilateral is equal, then it is a parallelogram.*

*Given.* A quadrilateral ABCD in which \( \angle A = \angle C \) and \( \angle B = \angle D \).
To prove. ABCD is a parallelogram.

Proof.

<table>
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</thead>
<tbody>
<tr>
<td>1. ( \angle A = \angle C )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \angle B = \angle D )</td>
<td>2. Given</td>
</tr>
<tr>
<td>3. ( \angle A + \angle B = \angle C + \angle D )</td>
<td>3. Adding 1 and 2</td>
</tr>
<tr>
<td>4. ( \angle A + \angle B + \angle C + \angle D = 360^\circ )</td>
<td>4. Sum of angles of a quadrilateral.</td>
</tr>
<tr>
<td>5. ( 2(\angle A + \angle B) = 360^\circ )</td>
<td>5. Using 3</td>
</tr>
<tr>
<td>( \Rightarrow \angle A + \angle B = 180^\circ )</td>
<td>Sum of co-interior angles = 180°, formed by lines BC, AD and transversal AB.</td>
</tr>
</tbody>
</table>

Similarly, AB \parallel DC
Hence, ABCD is a parallelogram.

**Theorem 12.6**

*If a pair of opposite sides of a quadrilateral is equal and parallel, then it is a parallelogram.*

**Given.** A quadrilateral ABCD in which AB \parallel DC and AB = DC.

**To prove.** ABCD is a parallelogram.

**Construction.** Join AC.

**Proof.**

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<td></td>
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<tr>
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<td>1. Alt. ( \angle )s, AB \parallel DC and AC is a transversal.</td>
</tr>
<tr>
<td>2. AB = DC</td>
<td>2. Given</td>
</tr>
<tr>
<td>3. AC = CA</td>
<td>3. Common</td>
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<tr>
<td>4. ( \triangle ABC \equiv \triangle CDA )</td>
<td>4. SAS rule of congruency.</td>
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<td>5. ( \angle ACB = \angle CAD )</td>
<td>5. c.p.c.t.</td>
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<tr>
<td>( \Rightarrow AD \parallel BC )</td>
<td>Alt. ( \angle )s are equal formed by lines AD, BC and transversal AC.</td>
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Hence, ABCD is a parallelogram.

**Theorem 12.7**

*The diagonals of a parallelogram bisect each other.*

**Given.** A parallelogram ABCD whose diagonals AC and BD intersect at O.

**To prove.** OA = OC and OB = OD.

**Proof.**

<table>
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<tr>
<td>In ( \triangle OAB ) and ( \triangle OCD )</td>
<td></td>
</tr>
<tr>
<td>1. ( \angle BAC = \angle ACD )</td>
<td>1. Alt. ( \angle )s, AB \parallel DC and AC is a transversal.</td>
</tr>
</tbody>
</table>
2. \( \angle ABD = \angle CDB \)  
2. Alt. \( \angle s \), AB \( \parallel \) DC and BD is a transversal.

3. \( AB = CD \)  
3. Opp. sides of a parallelogram are equal.

4. \( \triangle OAB \equiv \triangle OCD \)  
\[ \therefore OA = OC \text{ and } OB = OD \]  
4. ASA rule of congruency  
c.p.c.t.

The converse of the above theorem is also true. In fact, we have:

**Theorem 12.8**

*If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.*

**Given.** A quadrilateral \( ABCD \) whose diagonals \( AC \) and \( BD \) intersect at \( O \) such that \( OA = OC \) and \( OB = OD \).

**To prove.** \( ABCD \) is a parallelogram.

**Proof.**

<table>
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<tbody>
<tr>
<td>In ( \triangle OAB ) and ( \triangle OCD )</td>
<td></td>
</tr>
<tr>
<td>1. ( OA = OC )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( OB = OD )</td>
<td>2. Given</td>
</tr>
<tr>
<td>3. ( \angle AOB = \angle COD )</td>
<td>3. Vert. opp. ( \angle s )</td>
</tr>
<tr>
<td>4. ( \triangle OAB \equiv \triangle OCD )</td>
<td>4. SAS rule of congruency</td>
</tr>
<tr>
<td>5. ( \angle OAB = \angle OCD )</td>
<td>5. c.p.c.t.</td>
</tr>
<tr>
<td>( \Rightarrow \angle CAB = \angle ACD )</td>
<td>Alt. ( \angle s ) are equal formed by lines AB, DC and transversal AC.</td>
</tr>
<tr>
<td>( \Rightarrow AB \parallel DC )</td>
<td></td>
</tr>
<tr>
<td>6. ( AB = CD )</td>
<td>6. c.p.c.t.</td>
</tr>
<tr>
<td>7. ( ABCD ) is a parallelogram.</td>
<td>7. In quadrilateral ( ABCD ), ( AB \parallel DC ) and ( AB = CD ) (Theorem 12.6)</td>
</tr>
</tbody>
</table>

- **Properties of a rectangle**
  Since every rectangle is a parallelogram, therefore, it has all the properties of a parallelogram. Additional properties of a rectangle are:
  - *All the (interior) angles of a rectangle are right angles.*
    In the adjoining figure, \( \angle A = \angle B = \angle C = \angle D = 90° \).
  - *The diagonals of a rectangle are equal.*
    In the adjoining figure, \( AC = BD \).

- **Properties of a rhombus**
  Since every rhombus is a parallelogram, therefore, it has all the properties of a parallelogram. Additional properties of a rhombus are:
  - *All the sides of a rhombus are equal.*
    In the adjoining figure, \( AB = BC = CD = DA \).
  - *The diagonals of a rhombus intersect at right angles.*
    In the adjoining figure, \( AC \perp BD \).
• The diagonals bisect the angles of a rhombus.
In the adjoining figure, diagonal AC bisects \( \angle A \) as well as \( \angle C \) and diagonal BD bisects \( \angle B \) as well as \( \angle D \).

Properties of a square
Since every square is a parallelogram, therefore, it has all the properties of a parallelogram. Additional properties of a square are:

• All the interior angles of a square are right angles.
In the adjoining figure, \( \angle A = \angle B = \angle C = \angle D = 90^\circ \).

• All the sides of a square are equal.
In the adjoining figure, \( AB = BC = CD = DA \).

• The diagonals of a square are equal.
In the adjoining figure, \( AC = BD \).

• The diagonals of a square intersect at right angles.
In the adjoining figure, \( AC \perp BD \).

• The diagonals bisect the angles of a square.
In the adjoining figure, diagonal AC bisects \( \angle A \) as well as \( \angle C \) and diagonal BD bisects \( \angle B \) as well as \( \angle D \).

In fact, a square is a rectangle as well as a rhombus, so it has all the properties of a rectangle as well as that of a rhombus.

Illustrative Examples

Example 1. If angles A, B, C and D of a quadrilateral ABCD, taken in order, are in the ratio 3 : 7 : 6 : 4 then ABCD is a trapezium. Is this statement true? Give reason for your answer.

Solution. As the angles are in the ratio 3 : 7 : 6 : 4, let these angles be 3x, 7x, 6x and 4x.
Since sum of angles of a quadrilateral is 360°,
\[ 3x + 7x + 6x + 4x = 360^\circ \]
\[ \Rightarrow 20x = 360^\circ \Rightarrow x = 18^\circ. \]
\[ \therefore \text{The angles are: } \angle A = 3 \times 18^\circ = 54^\circ, \]
\[ \angle B = 7 \times 18^\circ = 126^\circ, \angle C = 6 \times 18^\circ = 108^\circ \text{ and } \]
\[ \angle D = 4 \times 18^\circ = 72^\circ. \]
We note that \( \angle A + \angle B = 54^\circ + 126^\circ = 180^\circ \).
Thus, the sum of co-interior angles is 180° formed by lines AD, BC and transversal AB, therefore, AD \parallel BC. So, ABCD is a trapezium. Hence, the given statement is true.

Example 2. Three angles of a quadrilateral ABCD are equal. Is it a parallelogram? Why or why not?

Solution. It need not be a parallelogram; because we may have \( \angle A = \angle B = \angle C = 80^\circ \), then \( \angle D = 360^\circ - 3 \times 80^\circ = 120^\circ \), so \( \angle B \neq \angle D \) (opposite angles are not equal).

Example 3. In a quadrilateral ABCD, AO and BO are the bisectors of \( \angle A \) and \( \angle B \) respectively. Prove that \( \angle AOB = \frac{1}{2} (\angle C + \angle D) \).

Solution. Given ABCD is a quadrilateral, OA and OB are the bisectors of \( \angle A \) and \( \angle B \) respectively. Mark the angles as shown in the figure given below.
As OA and OB are bisectors of \(\angle A\) and \(\angle B\) respectively,
\[
\angle 1 = \frac{1}{2} \angle A \quad \text{and} \quad \angle 2 = \frac{1}{2} \angle B \quad \text{(i)}
\]
\[
\angle AOB + \angle 1 + \angle 2 = 180^\circ \quad \text{(sum of angles in \(\triangle OAB\))}
\]
\[\Rightarrow \angle AOB = 180^\circ - (\angle 1 + \angle 2) \]
\[\Rightarrow \angle AOB = 180^\circ - \frac{1}{2} (\angle A + \angle B) \quad \text{(using (i))}
\]
\[\Rightarrow \angle AOB = 180^\circ - \frac{1}{2} (360^\circ - (\angle C + \angle D)) \]
\[\Rightarrow \angle AOB = \frac{1}{2} (\angle C + \angle D). \]

**Example 4.** Diagonals of a quadrilateral \(ABCD\) bisect each other. If \(\angle A = 45^\circ\), determine \(\angle B\).

**Solution.** Since the diagonals AC and BD of quadrilateral \(ABCD\) bisect each other, \(ABCD\) is a parallelogram.

\(AD \parallel BC\) and \(AB\) is a transversal,
\[
\angle A + \angle B = 180^\circ \quad \text{(sum of co-interior angles = 180°)}
\]
\[\Rightarrow 45^\circ + \angle B = 180^\circ \]
\[\Rightarrow \angle B = 135^\circ. \]

**Example 5.** The angle between two altitudes of a parallelogram through the vertex of an obtuse angle of the parallelogram is 60°. Find the angles of the parallelogram.

**Solution.** Let \(ABCD\) be parallelogram such that
\[DM \perp AB, \; DN \perp BC\] and \(\angle MDN = 60^\circ.\)

In quadrilateral \(DMBN,\)
\[
\angle MDN + \angle M + \angle N + \angle B = 360^\circ
\]
\[\Rightarrow 60^\circ + 90^\circ + 90^\circ + \angle B = 360^\circ \]
\[\Rightarrow \angle B = 120^\circ. \]

\(AD \parallel BC\) and \(AB\) is a transversal,
\[
\angle A + \angle B = 180^\circ
\]
\[\Rightarrow \angle A + 120^\circ = 180^\circ \Rightarrow \angle A = 60^\circ.
\]
\[
\angle C = \angle A \quad \text{and} \quad \angle D = \angle B
\]
\[\Rightarrow \angle C = 60^\circ \quad \text{and} \quad \angle D = 120^\circ. \]

Hence, the angles of \(ABCD\) are 60°, 120°, 60°, 120°.

**Example 6.** In the adjoining figure, \(ABCD\) is parallelogram. Find the values of \(x, y\) and \(z\).

**Solution.** Given \(ABCD\) is a parallelogram,
\[
3x - 1 = 2x + 2 \quad \text{(opp. sides are equal)}
\]
\[\Rightarrow x = 3. \]
\[
\angle D = \angle B = 102^\circ \quad \text{(opp. \(\angle s\) are equal)}
\]
For \(\triangle ACD,\)
\[
y = 50^\circ + \angle D \quad \text{\(\because\) ext. \(\angle = \) sum of two int. opp. \(\angle s\)}
\]
\[\Rightarrow y = 50^\circ + 102^\circ = 152^\circ \quad \text{(AD \parallel BC, sum of co-int. \(\angle s = 180^\circ\))}
\]
\[\Rightarrow \angle DAB + 102^\circ = 180^\circ \]
\[\Rightarrow \angle DAB = 180^\circ - 102^\circ = 78^\circ. \]

From figure, \(z = \angle DAB - \angle DAC = 78^\circ - 50^\circ = 28^\circ.\)
Example 7. In the adjoining figure, ABCD is a parallelogram. Find the ratio of AB : BC. All measurements are in centimetres.

Solution. Given ABCD is a parallelogram,

\[ 3x - 4 = y + 5 \]  
\[ \Rightarrow 3x - y - 9 = 0 \]  \( \text{(i)} \)
and \[ 2x + 5 = y - 1 \]  
\[ \Rightarrow 2x - y + 6 = 0 \]  \( \text{(ii)} \)

On subtracting \( \text{(ii)} \) from \( \text{(i)} \), we get

\[ x - 15 = 0 \Rightarrow x = 15. \]

On substituting this value of \( x \) in \( \text{(i)} \), we get

\[ 3 \times 15 - y - 9 = 0 \Rightarrow 36 - y = 0 \Rightarrow y = 36. \]

\[ \therefore AB = 3x - 4 = 41 \]
and \[ BC = y - 1 = 35. \]

Hence, \( AB : BC = 41 : 35 \).

Example 8. In a rectangle ABCD, diagonals intersect at O. If \( \angle OAB = 30^\circ \), find

(i) \( \angle ACB \)  
(ii) \( \angle ABO \)  
(iii) \( \angle COD \)  
(iv) \( \angle BOC \).

Solution. 

(i) \( \angle ABC = 90^\circ \)  \( \text{(each angle of a rectangle = 90^\circ)} \)
\[ \Rightarrow \angle ACB = 180^\circ - 30^\circ - 90^\circ = 60^\circ. \]

(ii) \( AC = BD \)  \( \text{(diagonals are equal)} \)
\[ \Rightarrow 2AO = 2BO \]  \( \text{(diagonals bisect each other)} \)
\[ \Rightarrow AO = BO \]
\[ \Rightarrow \angle ABO = \angle OAB \]  \( \text{(angles opp. equal sides in \( \triangle OAB \)} \)
\[ \Rightarrow \angle ABO = 30^\circ \]  \( \text{\( \because \angle OAB = 30^\circ \) given} \)

(iii) \( \angle AOB + 30^\circ + 30^\circ = 180^\circ \)
\[ \Rightarrow \angle AOB = 180^\circ - 30^\circ - 30^\circ = 120^\circ. \]

But \( \angle COD = \angle AOB \)
\[ \Rightarrow \angle COD = 120^\circ. \]

(iv) \( \angle BOC + 120^\circ = 180^\circ \)
\[ \Rightarrow \angle BOC = 180^\circ - 120^\circ = 60^\circ. \]

Example 9. In the adjoining figure, ABCD is a square and CDE is an equilateral triangle. Find

(i) \( \angle AED \)  
(ii) \( \angle EAB \)  
(iii) reflex \( \angle AEC \).

Solution. (i) From figure, \( \angle ADE = 90^\circ - 60^\circ \)
\[ \Rightarrow \angle ADE = 30^\circ. \]
\[ ED = DC \]  \( \text{(sides of equilateral triangle)} \)
\[ AD = DC \]  \( \text{(sides of square)} \)
\[ \Rightarrow ED = AD \Rightarrow \angle AED = \angle EAD \]  \( \text{(angles opp. equal sides in \( \triangle AED \)} \)
But \( \angle AED + \angle EAD + \angle ADE = 180^\circ \)
\[ \Rightarrow 2\angle AED + 30^\circ = 180^\circ \]
\[ \Rightarrow \angle AED = \frac{180^\circ - 30^\circ}{2} = 75^\circ. \]
(ii) \( \angle EAB = 90° - 75° = 15° \)

\( \therefore \angle EAD = \angle AED = 75° \)

(iii) Reflex \( \angle AEC = 360° - 75° - 60° = 225° \).

Example 10. BEC is an equilateral triangle in the square ABCD. Find the value of \( x \) in the figure.

Solution. Since ABCD is a square and BD is a diagonal,
\( \therefore \angle DBC = 45° \).
As BEC is an equilateral triangle,
\( \angle BCE = 60° \).
In \( \triangle BFC, x + 45° + 60° = 180° \)
\( \Rightarrow x = 180° - 45° - 60° = 75° \).

Example 11. In the adjoining figure, ABCD is a rhombus and ABE is an equilateral triangle. E and D lie on opposite sides of AB. If \( \angle BCD = 78° \), calculate \( \angle ADE \) and \( \angle BDE \).

Solution. Since ABCD is a rhombus, \( \angle DAB = \angle BCD = 78° \).
As ABE is an equilateral triangle, \( \angle BAE = 60° \).
From figure,
\( \angle DAE = \angle DAB + \angle BAE = 78° + 60° = 138° \).
Also \( BA = AE \) (Since ABE is equilateral triangle)
and \( BA = AD \) (\( \therefore \) ABCD is a rhombus)
\( \Rightarrow \ AE = AD \Rightarrow \angle ADE = \angle AED \) (\( \therefore \) angles opp. equal sides in \( \triangle AED \))
\( \therefore \angle ADE = \frac{1}{2} (180° - 138°) = \frac{1}{2} \times 42° = 21° \).

In \( \triangle ABCD, BC = CD \) (\( \therefore \) ABCD is a rhombus)
\( \Rightarrow \angle CBD = \angle CDB \).
\( \therefore \angle CBD = \frac{1}{2} (180° - 78°) = \frac{1}{2} (102°) = 51° \).
But \( \angle BDA = \angle CBD \) (BC \parallel AD, alt. \( \angle \)s are equal)
\( \Rightarrow \angle BDA = 51° \).
From figure, \( \angle BDE = \angle BDA - \angle EDA = 51° - 21° = 30° \).

Example 12. In parallelogram ABCD, \( AB = 10 \) cm and \( AD = 6 \) cm. The bisector of \( \angle A \) meets DC in E. AE and BC produced meet at F. Find the length of CF.

Solution. Mark the angles as shown in the figure.
As AE is bisector of \( \angle A \),
\( \angle 1 = \angle 2 \) \( \ldots (i) \)

Since ABCD is a parallelogram, AD \parallel BC i.e. AD \parallel BF
\( \therefore \angle 1 = \angle 3 \) (alt. \( \angle \)s are equal)
\( \Rightarrow \angle 2 = \angle 3 \) (using \( (i) \))
In \( \triangle ABF, \angle 2 = \angle 3 \)
\( \Rightarrow BF = AB \) (sides opp. equal angles are equal)
\( \Rightarrow BC + CF = 10 \) cm
\( \Rightarrow AD + CF = 10 \) cm (\( BC = AD \), opp. sides of a \( \parallel \ gm \))
\( \Rightarrow 6 \) cm + CF = 10 cm \( \Rightarrow CF = 4 \) cm.
Example 13. **ABCD is a rhombus in which altitude from D to side AB bisects AB. Find the angles of the rhombus.**

**Solution.** ABCD is a rhombus in which DM ⊥ AB such that M is mid-point of AB. Join BD.

In ∆DAM and ∆DBM,

\[
\begin{align*}
\angle AMD &= \angle BMD \\
AM &= BM \\
DM &= DM
\end{align*}
\]

(each = 90°)

(M is mid-point of AB)

(common)

∴ ∆DAM ≅ ∆DBM (SAS rule of congruency)

⇒ AD = BD

Also,

AD = AB (∵ ABCD is a rhombus)

⇒ AD = BD = AB

⇒ ∆ABD is an equilateral triangle

⇒ ∠A = 60°.

Now, AD || BC and AB is a transversal,

∴ ∠A + ∠B = 180° (sum of co-interior ∠s)

⇒ 60° + ∠B = 180° ⇒ ∠B = 120°

∠C = ∠A and ∠D = ∠B (opp. ∠s in a || gm are equal)

⇒ ∠C = 60° and ∠D = 120°.

Hence, the angles of the rhombus are 60°, 120°, 60°, 120°.

Example 14. **Prove that the diagonals of a rectangle are equal.**

**Solution.** Let ABCD be a rectangle. We need to prove that AC = BD.

In ∆ABC and ∆BAD,

\[
\begin{align*}
BC &= AD \\
AB &= BA \\
\angle ABC &= \angle BAD \\
\triangle ABC &= \triangle BAD
\end{align*}
\]

(opp. sides of a rectangle are equal)

(common)

(each angle of a rectangle = 90°)

(SAS rule of congruency)

∴ AC = BD (c.p.c.t.)

Example 15. **If the diagonals of a parallelogram are equal, then prove that it is a rectangle.**

**Solution.** Let ABCD be a parallelogram in which AC = BD. We need to prove that ∠A = 90°.

In ∆ABC and ∆BAD,

\[
\begin{align*}
BC &= AD \\
AB &= AB \\
AC &= BD \\
\triangle ABC &= \triangle BAD
\end{align*}
\]

(opp. sides of a || gm)

(common)

(given, diagonals are equal)

(SSS rule of congruency)

∴ ∠B = ∠A (c.p.c.t.)

As AD || BC and AB is a transversal,

\[
\begin{align*}
\angle A + \angle B &= 180° \\
\angle A + \angle A &= 180° \\
2\angle A &= 180° \\
\angle A &= 90°.
\end{align*}
\]

∴ ABCD is a rectangle.

Example 16. **Show that the diagonals of a rhombus bisect each other at right angles.**

**Solution.** Let ABCD be a rhombus and let its diagonals AC and BD meet at O.
We need to prove that $OA = OC$, $OB = OD$ and $\angle AOB = 90^\circ$. As every rhombus is a parallelogram, therefore, the diagonals bisect each other \textit{i.e.} $OA = OC$ and $OB = OD$.

Thus, the diagonals of a rhombus bisect each other.

In $\triangle OAB$ and $\triangle OCB$,
\[
\begin{align*}
OA &= OC \quad \text{(proved above)} \\
OB &= OB \quad \text{(common)} \\
AB &= BC \quad \text{(sides of a rhombus)} \\
\therefore \quad \triangle OAB &\cong \triangle OCB \quad \text{(by SSS rule of congruency)} \\
\therefore \quad \angle AOB &= \angle BOC \quad \text{(c.p.c.t.)}
\end{align*}
\]

But $\angle AOB + \angle BOC = 180^\circ$ \text{(linear pair)}
\[\Rightarrow \quad \angle AOB + \angle AOB = 180^\circ \Rightarrow \angle AOB = 90^\circ.\]

Hence, the diagonals of a rhombus bisect each other at right angles.

**Example 17.** If the diagonals of a quadrilateral bisect each other at right angles, then prove that the quadrilateral is a rhombus.

**Solution.** Let $ABCD$ be a quadrilateral in which the diagonals $AC$ and $BD$ bisect each other at $O$ and are at right angles \textit{i.e.} $OA = OC$, $OB = OD$ and $AC \perp BD$. We need to prove that $ABCD$ is a rhombus.

As the diagonals of the quadrilateral $ABCD$ bisect each other, $ABCD$ is a parallelogram.

In $\triangle OAB$ and $\triangle OCB$,
\[
\begin{align*}
OA &= OC \quad \text{(from given)} \\
OB &= OB \quad \text{(common)} \\
\angle AOB &= \angle COB \quad \text{(each = 90°, because } AC \perp BD \text{ given)} \\
\therefore \quad \triangle OAB &\cong \triangle OCB \quad \text{(by SAS rule of congruency)} \\
\therefore \quad AB &= BC \quad \text{(c.p.c.t.)}
\end{align*}
\]

Thus, $ABCD$ is a parallelogram in which two adjacent sides are equal, therefore, $ABCD$ is a rhombus.

**Example 18.** Prove that the diagonals of a square are equal and bisect each other at right angles.

**Solution.** Let $ABCD$ is a square and let its diagonals $AC$ and $BD$ meet at $O$. We need to prove that $OA = OC$, $OB = OD$, $\angle AOB = 90^\circ$ and $AC = BD$.

As $ABCD$ is a square, so it is a parallelogram. Therefore, its diagonals bisect each other \textit{i.e.} $OA = OC$ and $OB = OD$.

Thus, the diagonals of a square bisect each other.

In $\triangle OAB$ and $\triangle OCB$,
\[
\begin{align*}
OA &= OC \quad \text{(proved above)} \\
AB &= BC \quad \text{(sides of a square)} \\
OB &= OB \quad \text{(common)} \\
\therefore \quad \triangle OAB &\cong \triangle OCB \quad \text{(by SSS rule of congruency)} \\
\therefore \quad \angle AOB &= \angle BOC \quad \text{(c.p.c.t.)}
\end{align*}
\]

But $\angle AOB + \angle BOC = 180^\circ$ \text{(linear pair)}
\[\Rightarrow \quad \angle AOB + \angle AOB = 180^\circ \Rightarrow \angle AOB = 90^\circ.\]

As $ABCD$ is a square, so one of its angle is $90^\circ$. Let $\angle A = 90^\circ$. 

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Now AD \parallel BC and AB is a transversal
\[ \angle A + \angle B = 180^\circ \]  
(sum of co-int. \( \angle \)s)
\[ \Rightarrow 90^\circ + \angle B = 180^\circ \Rightarrow \angle B = 90^\circ. \]

In \( \triangle ABC \) and \( \triangle BAD \),
- \( BC = AD \)  
  (sides of a square)
- \( AB = BA \)  
  (common)
- \( \angle B = \angle A \)  
  (each = 90°)
\[ \therefore \quad \triangle ABC \equiv \triangle BAD \]
\[ \therefore \quad AC = BD \]  
(c.p.c.t.)

Hence, the diagonals of a square are equal and bisect each other at right angles.

**Example 19.** In the adjoining figure, \( ABCD \) is a parallelogram and \( AP, CQ \) are perpendiculars from the vertices \( A, C \) respectively on the diagonal \( BD \). Show that
(i) \( \triangle APB \equiv \triangle CQD \)
(ii) \( AP = CQ \).

**Solution.**
(i) In \( \triangle APB \) and \( \triangle CQD \),
- \( AB = CD \)  
  (opp. sides of a \( \parallel \) gm)
- \( \angle P = \angle Q \)  
  (each = 90°)
- \( \angle APB = \angle CDQ \)  
  (alt. \( \angle \)s, \( AB \parallel DC \) and \( BD \) is a transversal)
\[ \therefore \quad \triangle APB \equiv \triangle CQD \]  
(by AAS rule of congruency)
\[ \therefore \quad AP = CQ \]  
(c.p.c.t.)

(ii) \( AP = CQ \)

**Example 20.** If \( E \) and \( F \) are points on diagonal \( AC \) of a parallelogram \( ABCD \) such that \( AE = CF \), then show that \( BFDE \) is a parallelogram.

**Solution.** As \( ABCD \) is a parallelogram, its diagonals bisect each other i.e. \( OA = OC \) and \( OB = OD \).

Given \( \quad AE = CF \)
\[ \therefore \quad OA - AE = OC - CF \]
\[ \Rightarrow \quad OE = OF. \]

Thus, in quadrilateral \( BFDE \), \( OE = OF \) and \( OB = OD \) i.e. its diagonals \( EF \) and \( BD \) bisect each other, therefore, \( BFDE \) is a parallelogram.

**Example 21.** \( ABCD \) is a parallelogram. If the bisectors of \( \angle A \) and \( \angle C \) meet the diagonal \( BD \) at \( P \) and \( Q \) respectively, prove that the quadrilateral \( PCQA \) is a parallelogram.

**Solution.**
**Given.** \( ABCD \) is a \( \parallel \) gm, \( AP \) bisects \( \angle A \) and \( CQ \) bisects \( \angle C \).

**To prove.** \( AP \parallel QC \) and \( PC \parallel AQ \).

**Construction.** Join \( AC \).

**Proof.**

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
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<tbody>
<tr>
<td>1. ( \angle BAP = \frac{1}{2} \angle A )</td>
<td>1. ( AP ) is bisector of ( \angle A ).</td>
</tr>
<tr>
<td>2. ( \angle DCQ = \frac{1}{2} \angle C )</td>
<td>2. ( CQ ) is bisector of ( \angle C ).</td>
</tr>
<tr>
<td>3. ( \angle BAP = \angle DCQ )</td>
<td>3. ( \angle A = \angle C ), since ( ABCD ) is a ( \parallel ) gm.</td>
</tr>
</tbody>
</table>
4. \( \angle BAC = \angle DCA \)  
4. Alt. \( \angle s \), since \( AB \parallel DC \).

5. \( \angle BAP - \angle BAC = \angle DCQ - \angle DCA \)  
5. Subtracting 4 from 3.

6. \( \angle CAP = \angle ACQ \)  
6. From figure.

7. \( AP \parallel QC \)  
Similarly, \( PC \parallel AQ \).  
Hence, \( PCQA \) is a parallelogram.  
Q.E.D

Example 22. Show that the bisectors of the angles of a parallelogram form a rectangle.

Solution. Let \( ABCD \) be a parallelogram and let \( P, Q, R \) and \( S \) be the points of intersection of the bisectors of \( \angle A \) and \( \angle B \), \( \angle B \) and \( \angle C \), \( \angle C \) and \( \angle D \), and \( \angle D \) and \( \angle A \) respectively. We need to show that \( PQRST \) is a rectangle.

As \( ABCD \) is a || gm, \( AD \parallel BC \) and \( AB \) is a transversal. 
\[ \therefore \quad \angle A + \angle B = 180^\circ \text{ (sum of co-int. } \angle s = 180^\circ) \]

\[ \Rightarrow \quad \frac{1}{2} \angle A + \frac{1}{2} \angle B = 90^\circ \]

\[ \Rightarrow \quad \angle PAB + \angle PBA = 90^\circ \quad (\therefore \text{ AP is bisector of } \angle A \text{ and BP is bisector of } \angle B) \]

In \( \triangle PAB \), \( \angle APB + \angle PAB + \angle PBA = 180^\circ \) (sum of angles in a \( \triangle \))

\[ \Rightarrow \quad \angle APB + 90^\circ = 180^\circ \Rightarrow \angle APB = 90^\circ \Rightarrow \angle SPQ = 90^\circ. \]

Similarly, \( \angle PQR = 90^\circ, \angle QRS = 90^\circ \) and \( \angle RSP = 90^\circ. \)

Thus, \( PQRST \) is a quadrilateral in which each angle is \( 90^\circ. \)

Now, \( \angle SPQ = \angle QRS \) (each = \( 90^\circ \))
and \( \angle PQR = \angle RSP \) (each = \( 90^\circ \))

Thus, \( PQRST \) is a quadrilateral in which both pairs of opposite angles are equal, therefore, \( PQRST \) is a parallelogram. Also, in this parallelogram one angle (in fact all angles) is \( 90^\circ. \)

Therefore, \( PQRST \) is a rectangle.

Example 23. In the adjoining figure, \( ABCD \) is a parallelogram. If \( AB = 2AD \) and \( P \) is mid-point of \( AB \), prove that \( \angle DPC = 90^\circ. \)

Solution. Given \( P \) is mid-point of \( AB \)

\[ \Rightarrow \quad AP = PB = \frac{1}{2} \text{ AB.} \]

Also \( AB = 2AD \Rightarrow AD = \frac{1}{2} \text{ AB.} \)
\[ \therefore \quad AP = AD. \]

In \( \triangle APD, AP = AD \)  
(angles opp. equal sides)

But \( \angle A + \angle ADP + \angle APD = 180^\circ \) (sum of angles in a \( \triangle \) = \( 180^\circ \))

\[ \Rightarrow \quad \angle A + \angle ADP + \angle APD = 180^\circ \quad (\therefore \angle ADP = \angle APD) \]

\[ \Rightarrow \quad 2 \angle ADP = 180^\circ - \angle A \Rightarrow \angle APD = \frac{180^\circ - \angle A}{2} \quad \ldots(i) \]

As \( PB = AP \) and \( BC = AD \) (opp. sides of || gm \( ABCD \))

\[ \Rightarrow \quad PB = BC. \]
In \(\triangle BPC\), \(PB = BC \Rightarrow \angle CPB = \angle BCP\).

But \(\angle B + \angle CPB + \angle BCP = 180^\circ\)

\[ \Rightarrow \angle B + \angle CPB + \angle CPB = 180^\circ \]

\[ \Rightarrow 2 \angle CPB = 180^\circ - \angle B \Rightarrow \angle CPB = \frac{180^\circ - \angle B}{2} \quad \text{...(ii)} \]

Adding (i) and (ii), we get

\[ \angle APD + \angle CPB = 180^\circ - \frac{1}{2} (\angle A + \angle B) \]

\[ = 180^\circ - \frac{1}{2} (180^\circ) \quad (\because \text{ABCD is a } ||\text{gm}, \text{AD} \parallel \text{BC}, \text{so } \angle A + \angle B = 180^\circ) \]

\[ \Rightarrow \angle APD + \angle CPB = 90^\circ \quad \text{...(iii)} \]

But \(\angle APD + \angle DPC + \angle CPB = 180^\circ \quad (\because \text{APB is a line})

\[ \Rightarrow (\angle APD + \angle CPB) + \angle DPC = 180^\circ \]

\[ \Rightarrow 90^\circ + \angle DPC = 180^\circ \]

\[ \Rightarrow \angle DPC = 90^\circ \quad \text{(using (iii))} \]

**Example 24.** In the parallelogram ABCD, M is mid-point of AC, and X, Y are points on AB and DC respectively such that AX = CY. Prove that

(i) triangle AXM is congruent to triangle CYM.

(ii) XMY is a straight line.

**Given.** ABCD is a ||gm, M is mid-point of AC, X, Y are points on AB, CD such that AX = CY.

**To prove.**

(i) \(\triangle AXM \cong \triangle CYM\)

(ii) XMY is a straight line.

**Construction.** Join XM and MY.

**Proof.**

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
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<tbody>
<tr>
<td>In (\triangle AXM) and (\triangle CYM)</td>
<td>1. Given.</td>
</tr>
<tr>
<td>1. AX = CY</td>
<td>1. Given.</td>
</tr>
<tr>
<td>2. AM = MC</td>
<td>2. M is mid-point of AC.</td>
</tr>
<tr>
<td>3. (\angle XAM = \angle MCY) (\therefore (i) \triangle AXM \cong \triangle CYM)</td>
<td>3. (\text{Alt. } \angle\text{s, since AB} \parallel \text{DC}). SAS rule of congruency.</td>
</tr>
<tr>
<td>4. (\angle CMY = \angle AMX)</td>
<td>4. (\text{‘c.p.c.t.’})</td>
</tr>
<tr>
<td>5. (\angle XMC = \angle XAM + \angle AXM)</td>
<td>5. Ext. (\angle) = sum of two int. opp. (\angle)s.</td>
</tr>
<tr>
<td>6. (\angle CMY + \angle XMC) (= \angle AMX + \angle XAM + \angle AXM)</td>
<td>6. Adding 4 and 5.</td>
</tr>
<tr>
<td>7. (\angle CMY + \angle XMC = 180^\circ) (\therefore ) XMY is a straight line</td>
<td>7. Sum of (\angle)s of a (\Delta = 180^\circ). Sum of adj. (\angle)s = 180°.</td>
</tr>
</tbody>
</table>

**Example 25.** In the adjoining figure, ABCD is a kite in which \(AB = AD\) and \(BC = CD\). Prove that:

(i) AC is a bisector of \(\angle A\) and of \(\angle C\).

(ii) AC is perpendicular bisector of BD.

**Solution.**

(i) In \(\triangle ABC\) and \(\triangle ADC\), \(AB = AD\) (given)
BC = CD
CA = CA
\[\therefore \triangle ABC \cong \triangle ADC \]  (given)
\[\angle BAC = \angle CAD \text{ and } \angle BCA = \angle ACD. \]  (common)
Hence, AC is bisector of \(\angle A\) and of \(\angle C\).

(ii) In \(\triangle OBC\) and \(\triangle ODC\),
\[\begin{align*}
BC &= CD \\
\angle BCO &= \angle OCD \\
OC &= OC
\end{align*} \]  (given)
\[\therefore \triangle OBC \cong \triangle ODC \]  (proved above)
\[\text{But } \angle BOC + \angle COD = 180^\circ \]  (sum of angles in \(\triangle OCD\))
\[\Rightarrow 2\angle BOC = 180^\circ \Rightarrow \angle BOC = 90^\circ. \]  (linear pair)
Hence, AC is perpendicular bisector of BD.

Example 26. In the adjoining kite, diagonals intersect at O.
If \(\angle ABO = 25^\circ\) and \(\angle OCD = 40^\circ\), find
(i) \(\angle ABC\)  (ii) \(\angle ADC\)  (iii) \(\angle BAD\).

Solution. (i) Since the diagonal BD bisects \(\angle ABC\),
\[\angle ABC = 2 \times \angle ABO = 2 \times 25^\circ = 50^\circ. \]
(ii) \(\angle DOC = 90^\circ\) (diagonals intersect at right angles)
\[\angle ODC + 40^\circ + 90^\circ = 180^\circ \]  (sum of angles in \(\triangle OCD\))
\[\Rightarrow \angle ODC = 180^\circ - 40^\circ - 90^\circ = 50^\circ. \]
Since the diagonal BD bisects \(\angle ADC\),
\[\angle ADC = 2 \times \angle ODC = 2 \times 50^\circ = 100^\circ. \]
(iii) Since the diagonal BD bisects \(\angle ADC\), \(\angle ADB = \angle ODC = 50^\circ.\)
\[\angle BAD + 50^\circ + 25^\circ = 180^\circ \]  (sum of angles in \(\triangle ABD\))
\[\Rightarrow \angle BAD = 180^\circ - 50^\circ - 25^\circ = 105^\circ. \]

Example 27. In the adjoining figure, ABCD is an isosceles trapezium and its diagonals meet at O. Prove that:
(i) \(\angle A = \angle B\) and \(\angle C = \angle D\).
(ii) \(AC = BD\).
(iii) \(OA = OB\) and \(OC = OD\).

Solution. (i) From C and D, draw perpendicul ars CN and DM on AB respectively.
In \(\triangle AMD\) and \(\triangle ABNC\),
\[\begin{align*}
AD &= BC \quad \text{(given)} \\
\angle AMD &= \angle CNB \\
\text{(\therefore DM} \perp \text{AB and CN} \perp \text{AB, by construction)} \\
MD &= CN \quad \text{(distance between parallel lines)} \\
\therefore &\quad \triangle AMD \cong \triangle ABNC \quad \text{(RHS rule of congruency)}
\end{align*} \]
\[\therefore \angle A = \angle B \quad \text{(c.p.c.t.)} \]
Also \(\angle A + \angle D = 180^\circ\) and \(\angle B + \angle C = 180^\circ\)  (\(\therefore AB \parallel DC\), sum of co-int. \(\angle s = 180^\circ\))
\[\Rightarrow \angle A + \angle D = \angle B + \angle C \]
\[ \angle D = \angle C \] 
\[ (i) \text{In} \Delta ABD \text{and} \Delta BAC, \]
\[ \angle A = \angle B \]
\[ AD = BC \]
\[ AB = AB \]
\[ \therefore \Delta ABD \cong \Delta BAC \]  
\[ \therefore AC = BD \]  
\[ (ii) \text{In} \Delta DAB \text{and} \Delta DBC, \]
\[ \angle A = \angle B \]  
\[ \text{given} \]
\[ \angle ADO = \angle BCO \]  
\[ (\because \Delta ABD \cong \Delta BAC, \text{so} \angle ADB = \angle ACB) \]
\[ \therefore \Delta DAO \cong \Delta OBC \]  
\[ \therefore OA = OB \text{and} OC = OD. \]  

Example 28. In the adjoining figure, ABCD is a trapezium. If \( \angle AOB = 126^\circ \) and \( \angle PDC = \angle QCD = 52^\circ \), find the values of \( x \) and \( y \).

Solution. Produce AP and BQ to meet at R.

In \( \Delta DRC \), \( \angle DRC = \angle RCD \) (each angle = \( 52^\circ \))
\[ \therefore DR = CR \]  
\[ \text{(sides opp. equal angles are equal)} \]
\[ \angle RAB = \angle RDC \]  
\[ \text{(corres. \, \angle s, \, AB \parallel DC).} \]

Similarly, \( \angle RBA = \angle RCD \).
\[ \therefore \angle RAB = \angle RBA \]
\[ \Rightarrow AR = RB. \]
\[ \therefore AD = AR - DR = RB - CR = BC \]
\[ \Rightarrow ABCD \text{is an isosceles trapezium.} \]
\[ \therefore OA = OB \]  
\[ \text{(Example 27)} \]
\[ \Rightarrow \angle OAB = \angle OBA. \]
\[ \therefore \angle OAB = \frac{180^\circ - 126^\circ}{2} = 27^\circ \]
\[ \angle DAC = \angle DAB - \angle OAB = 52^\circ - 27^\circ = 25^\circ \]
\[ \therefore x = 25^\circ. \]
\[ \angle ACB + \angle CAB + \angle ABC = 180^\circ \]
\[ \Rightarrow y + 27^\circ + 52^\circ = 180^\circ \Rightarrow y = 180^\circ - 27^\circ - 52^\circ \]
\[ \Rightarrow y = 101^\circ. \]

Exercise 12.1

1. If two angles of a quadrilateral are 40° and 110° and the other two are in the ratio 3 : 4, find these angles.
2. If the angles of a quadrilateral, taken in order, are in the ratio 1 : 2 : 3 : 4, prove that it is a trapezium.
3. If an angle of a parallelogram is two-thirds of its adjacent angle, find the angles of the parallelogram.
4. (a) In figure (1) given below, ABCD is a parallelogram in which \( \angle DAB = 70^\circ, \angle DBC = 80^\circ \). Calculate angles CDB and ADB.