# 2 Exponents and Powers

## INTRODUCTION

In Class VII, you have learnt about powers with positive integers as exponents, expressing the natural numbers as a product of powers of prime factors, laws of exponents and expressing very large numbers in the standard form (*i.e.* powers of 10).

In this chapter, you will learn about powers with negative integers as exponents, law of exponents, expressing very small numbers in standard form using exponents and comparing very large and very small numbers.

## POWERS WITH NEGATIVE EXPONENTS

You know that $2^4 = 2 \times 2 \times 2 \times 2$ (2 is called the base and 4 is called the exponent)

$2^n = 2 \times 2 \times 2$ ..... $n$ times, $n$ is a natural number.

Let us now find $2^{-2}$ is equal to what?

Observe the following:

$2^4 = 2 \times 2 \times 2 \times 2 = 16$

$2^3 = 2 \times 2 \times 2 = 8 = \frac{16}{2}$

$2^2 = 2 \times 2 = 4 = \frac{8}{2}$

$2^1 = 2 = \frac{4}{2}$

$2^0 = 1 = \frac{2}{2}$.

Here you can see that as exponent decreases by 1, the previous number is divided by the base 2.

So, continuing the above pattern, we get

$2^{-1} = 1 \div 2 = \frac{1}{2}$

$2^{-2} = \frac{1}{2} \div 2 = \frac{1}{2 \times 2} = \frac{1}{2^2}$

$2^{-3} = \frac{1}{2^2} \div 2 = \frac{1}{2 \times 2 \times 2} = \frac{1}{2^3}$.

So we have $2^{-3} = \frac{1}{2^3}$ or $2^3 = \frac{1}{2^{-3}}$.

In general, we can say that for any non-zero integer $a$, $a^{-m} = \frac{1}{a^m}$ where $m$ is a positive integer.

$a^{-m}$ is called multiplicative inverse of $a^m$ and $a^m$ is called multiplicative inverse of $a^{-m}$.

We also have:

If $\frac{p}{q}$ is a non-zero rational number and $m$ is any positive integer, then

$$\left(\frac{p}{q}\right)^{-m} = \frac{1}{\left(\frac{p}{q}\right)^m} = \left(\frac{q}{p}\right)^m = \frac{q^m}{p^m}.$$

**Example 1.** Evaluate:

(*i*) $3^{-4}$ (*ii*) $\left(\frac{2}{3}\right)^{-5}$ (*iii*) $(-4)^{-2}$

**Solution.**

(*i*) $3^{-4} = \frac{1}{3^4} = \frac{1}{3\times3\times3\times3} = \frac{1}{81}$. $\left(\because a^{-m} = \frac{1}{a^m}\right)$

(*ii*) $\left(\frac{2}{3}\right)^{-5} = \left(\frac{3}{2}\right)^{5} = \frac{3^5}{2^5} = \frac{3\times3\times3\times3\times3}{2\times2\times2\times2\times2} = \frac{243}{32}$. $\left(\because \left(\frac{p}{q}\right)^{-m} = \left(\frac{q}{p}\right)^{m}\right)$

(*iii*) $(-4)^{-2} = \frac{1}{(-4)^2} = \frac{1}{(-4)\times(-4)} = \frac{1}{16}$.

**Example 2.** Find the multiplicative inverse of the following:

(*i*) $2^{-4}$ (*ii*) $7^{-2}$ (*iii*) $\left(\frac{4}{5}\right)^3$

**Solution.**

(*i*) As multiplicative inverse of $a^{-m} = a^m$

$\therefore$ multiplicative inverse of $2^{-4} = 2^4 = 2\times2\times2\times2 = 16$.

(*ii*) Multiplicative inverse of $7^{-2} = 7^2 = 7\times7 = 49$.

(*iii*) Multiplicative inverse of $\left(\frac{4}{5}\right)^3 = \left(\frac{4}{5}\right)^{-3} = \frac{1}{\left(\frac{4}{5}\right)^3}$

$= \left(\frac{5}{4}\right)^3 = \frac{5^3}{4^3} = \frac{5\times5\times5}{4\times4\times4} = \frac{125}{64}$.

**Example 3.** Write the following numbers in expanded form using exponents:

(*i*) 1025.63 (*ii*) 1256.249

**Solution.**

(*i*) We have

$1025.63 = 1\times1000 + 0\times100 + 2\times10 + 5\times1 + \frac{6}{10} + \frac{3}{100}$

$= 1\times10^3 + 0\times10^2 + 2\times10^1 + 5\times1 + 6\times10^{-1} + 3\times10^{-2}$

$\left(\because \frac{1}{10} = 10^{-1} \text{ and } \frac{1}{100} = \frac{1}{10\times10} = 10^{-2}\right)$

(*ii*) $1256.249 = 1\times1000 + 2\times100 + 5\times10 + 6\times1 + \frac{2}{10} + \frac{4}{100} + \frac{9}{1000}$

$= 1\times10^3 + 2\times10^2 + 5\times10^1 + 6\times1 + 2\times10^{-1} + 4\times10^{-2} + 9\times10^{-3}$.

## LAWS OF EXPONENTS

In previous class you have learnt that for any rational number $a$, $a^m \times a^n = a^{m+n}$, where $m$ and $n$ are positive integers. Does this law also hold for negative exponents? Let us explore. Observe the following:

(*i*) $2^{-3}\times2^{-4} = \frac{1}{2^3}\times\frac{1}{2^4} = \frac{1}{2^3\times2^4} = \frac{1}{2^{3+4}}$ $(\because a^m\times a^n = a^{m+n})$

$= \frac{1}{2^7} = 2^{-7} = 2^{(-3)+(-4)}$ (Note that $-7 = (-3) + (-4)$)

(*ii*) $5^{-2} \times 5^3 = \frac{1}{5^2} \times 5^3 = \frac{5^3}{5^2} = 5^{3-2} = 5^1$ $\left(\because \frac{a^m}{a^n} = a^{m-n},\ a \neq 0\right)$

$= 5^{(-2)+3}$ (Note that $-2 + 3 = 1$)

In general, we can say that:

If $a$ is any non-zero rational number and $m$, $n$ are integers, then

$$a^m \times a^n = a^{m+n}$$

Similarly, we can verify the remaining laws of exponents. So, we have:

If $a$, $b$ are two non-zero rational numbers and $m$, $n$ are any integers, then

(*i*) $a^m \times a^n = a^{m+n}$ (*ii*) $\frac{a^m}{a^n} = a^{m-n}$ (*iii*) $(a^m)^n = a^{mn}$

(*iv*) $(ab)^m = a^m \times b^m$ (*v*) $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ (*vi*) $a^0 = 1$.

**Example 4.** Simplify and write in exponential form:

(*i*) $(-2)^{-3} \times (-2)^{-4}$ (*ii*) $4^5 \div 4^{-6}$ (*iii*) $3^2 \times 3^{-5} \times 3^6$.

**Solution.**

(*i*) $(-2)^{-3} \times (-2)^{-4} = (-2)^{-3+(-4)} = (-2)^{-7}$ $(\because a^m \times a^n = a^{m+n})$

$= \frac{1}{(-2)^7}$ $\left(\because a^{-m} = \frac{1}{a^m}\right)$

(*ii*) $4^5 \div 4^{-6} = \frac{4^5}{4^{-6}} = 4^{5-(-6)} = 4^{11}$ $\left(\because \frac{a^m}{a^n} = a^{m-n}\right)$

(*iii*) $3^2 \times 3^{-5} \times 3^6 = 3^{2+(-5)} \times 3^6 = 3^{-3} \times 3^6$

$= 3^{-3+6} = 3^3$.

**Example 5.** Express $9^{-3}$ as a power with base 3.

**Solution.** We know that $9 = 3 \times 3 = 3^2$

$\therefore$ $9^{-3} = (3^2)^{-3} = 3^{2\times(-3)} = 3^{-6}$.

**Example 6.** Find $m$ so that $(-3)^{m+1} \times (-3)^5 = (-3)^7$.

**Solution.** $(-3)^{m+1} \times (-3)^5 = (-3)^7$

$\Rightarrow$ $(-3)^{m+1+5} = (-3)^7$

$\Rightarrow$ $(-3)^{m+6} = (-3)^7$.

Equating the exponents on both sides, we get

$m + 6 = 7$

$\Rightarrow$ $m = 7 - 6 = 1$.

**Example 7.** Simplify:

(*i*) $\left\{\left(\frac{1}{3}\right)^{-2} - \left(\frac{1}{2}\right)^{-3}\right\} \div \left(\frac{1}{4}\right)^{-2}$ (*ii*) $\left(\frac{5}{8}\right)^{-7} \times \left(\frac{8}{5}\right)^{-5}$

**Solution.**

(*i*) $\left\{\left(\frac{1}{3}\right)^{-2} - \left(\frac{1}{2}\right)^{-3}\right\} \div \left(\frac{1}{4}\right)^{-2} = \left\{\frac{1^{-2}}{3^{-2}} - \frac{1^{-3}}{2^{-3}}\right\} \div \frac{1^{-2}}{4^{-2}}$

$= \left\{\frac{3^2}{1^2} - \frac{2^3}{1^3}\right\} \div \frac{4^2}{1^2}$ $\left(\because a^{-m} = \frac{1}{a^m}\right)$

$= \{9 - 8\} \div 16 = 1 \times \frac{1}{16} = \frac{1}{16}$.

$(iii)\ \left(\frac{81}{256}\right)^{\frac{-3}{4}} = \left(\frac{3^4}{4^4}\right)^{-3/4} = \left\{\left(\frac{3}{4}\right)^4\right\}^{-3/4} = \left(\frac{3}{4}\right)^{4\times\left(\frac{-3}{4}\right)}$ $(\because (a^m)^n = a^{mn})$

$= \left(\frac{3}{4}\right)^{-3} = \left(\frac{4}{3}\right)^3$ $\left(\left(\frac{p}{q}\right)^{-m} = \left(\frac{q}{p}\right)^m\right)$

$= \frac{4^3}{3^3}$

$(iv)\ \left[\left\{\left(\frac{8}{9}\right)^{-2}\right\}^{\frac{1}{3}}\right]^6 = \left[\left(\frac{8}{9}\right)^{(-2)\times\frac{1}{3}}\right]^6 = \left[\left(\frac{8}{9}\right)^{\frac{-2}{3}}\right]^6$

$= \left(\frac{8}{9}\right)^{\left(\frac{-2}{3}\right)\times 6} = \left(\frac{8}{9}\right)^{(-2)\times 2}$

$= \left(\frac{8}{9}\right)^{-4} = \left(\frac{9}{8}\right)^4 = \left(\frac{3^2}{2^3}\right)^4 = \frac{3^{2\times 4}}{2^{3\times 4}} = \frac{3^8}{2^{12}}.$

**Example 10.** By what number should $\frac{-8}{25}$ be divided to get $\frac{-5}{2}$?

**Solution.** Let the required number be $x$, then

$\left(\frac{-8}{25}\right) \div x = \frac{-5}{2}$

$\Rightarrow \frac{-8}{25} \times \frac{1}{x} = -\frac{5}{2} \Rightarrow \frac{1}{x} = \frac{-5}{2} \div \frac{-8}{25}$

$\Rightarrow \frac{1}{x} = \frac{-5}{2} \times \frac{25}{-8} \Rightarrow \frac{1}{x} = \frac{(-5)\times 25}{2\times(-8)}$

$\Rightarrow \frac{1}{x} = \frac{-125}{-16} \Rightarrow \frac{1}{x} = \frac{125}{16}$

$\Rightarrow x = \frac{16}{125}.$

Hence, $\frac{-8}{25}$ should be divided by $\frac{16}{125}$ to get $\frac{-5}{2}$.

## Exercise 2.1

**1.** Evaluate:

$(i)\ \left(\frac{3}{5}\right)^{-2}$ $(ii)\ (-3)^{-3}$ $(iii)\ \left(\frac{2}{7}\right)^{-4}$ $(iv)\ \left(-\frac{1}{2}\right)^{-5}.$

**2.** Simplify:

$(i)\ [(2)^{-1} + (4)^{-1} + (3)^{-1}]^{-1}$ $(ii)\ [(4)^{-1} - (5)^{-1}]^2 \times \left(\frac{5}{8}\right)^{-1}$

$(iii)\ [4^0 + 4^2 - 2^3] \times 3^{-2}$ $(iv)\ \left[(5)^2 - \left(\frac{1}{4}\right)^{-2}\right] \times \left(\frac{3}{4}\right)^{-2}$

$(v)\ \left[6^{-1} - \left(\frac{1}{2}\right)^3\right] + \left[\frac{1}{4} - (2)^{-3}\right]$ $(vi)\ \left\{\left(\frac{1}{3}\right)^{-1} - \left(\frac{1}{4}\right)^{-1}\right\}^{-1}.$

**3.** Find the multiplicative inverse of the following:

$(i)\ \left(\frac{81}{16}\right)^{-3/4}$ $(ii)\ \left\{\left(\frac{-3}{2}\right)^{-4}\right\}^{\frac{1}{2}}$ $(iii)\ \left(\frac{5}{7}\right)^{-2} \times \left(\frac{5}{7}\right)^4 \div \left(\frac{5}{7}\right)^3$

**4.** $(i)$ Express $4^{-3}$ as a power with base 2.

$(ii)$ Express $125^{-4}$ as a power with base 5.

**5.** Write the following numbers in expanded form using exponents:

$(i)$ 2789.453 $(ii)$ 3007.805