8 Direct and Inverse Variation

INTRODUCTION

In Class VII, you have learnt that how to solve a real life simple problem by using unitary method. While solving the problems by unitary method, you must have noticed that increase or decrease (variation) in one quantity causes the increase or decrease (variation) in other quantity. In this chapter we shall learn about variation, its types and uses of variations in many day to day life situations.

VARIATION

As we know, if 2 pen cost ₹ 10, then 4 pen will cost ₹ 20 and 1 pen will cost ₹ 5.

It means, ***more pens, more cost***

less pens, less cost

So we can say that:

The cost of pens varies directly to the number of pens.

Again, if 2 men finish a job in 4 days, then 4 men can finish the same job in 2 days and 1 man alone can finish it in 8 days.

It means, ***more men at work, less time taken to finish the job***

less men at work, more time taken to finish the job

So we can say that:

The time taken to finish a job varies inversely to the number of men at work.

Two quantities are said to be in variation if any increase or decrease in one quantity causes the increase or decrease in other quantity.

There are two types of variation:

(*i*) *Direct variation*

(*ii*) *Inverse variation*.

DIRECT VARIATION

Two quantities are said to ***vary directly*** *if the increase (or decrease) in one quantity causes the increase (or decrease) in the other quantity*.

For example:

(*i*) The cost of articles varies directly to the number of articles.

More articles, more cost.

Less articles, less cost.

(*ii*) The work done varies directly to the working time.

More work done in more time.

Less work done in less time.

Let us consider the following:

If the cost of 2 litres of milk is ₹ 40, then the cost of 4 litres of milk will be ₹ 80.

Similarly, we can find the cost of 5 litres or 8 litres of milk.

Study the following table:

Milk in litres (x)	2	4	5	8	11
Cost in ₹ (y)	40	80	100	–	–

We find that as x increases, value of y also increases and $\frac{x}{y}$ is constant.

In this case, $\frac{x}{y} = \frac{2}{40} = \frac{4}{80} = \frac{5}{100} = \ldots.. = \frac{1}{20}$

i.e. $\frac{x}{y}$ = constant (k) or $x = ky$, where k is called **constant of variation**.

So we can say that:

Two quantities x and y are said to be in direct variation if $\frac{x}{y}$ *is constant.*

ACTIVITY 3

To deduce direct variation

Steps

1. Cut 5 circles of different radii from different coloured card sheets and paste them on a white chart paper. Give number to these circle as 1, 2, 3, 4, 5.

2. Mark the diameters of all the circles. Measure the diameters of all the circles with the help of ruler and note it down in the table shown below.

Circle	1	2	3	4	5
Diameter					
Circumference					

Materials required

(*i*) 5 different coloured card sheets

(*ii*) A white chart paper.

(*iii*) A pair of scissors

(*iv*) Gluestick

(*v*) Geometry box

(*vi*) Fine inelastic string/thread.

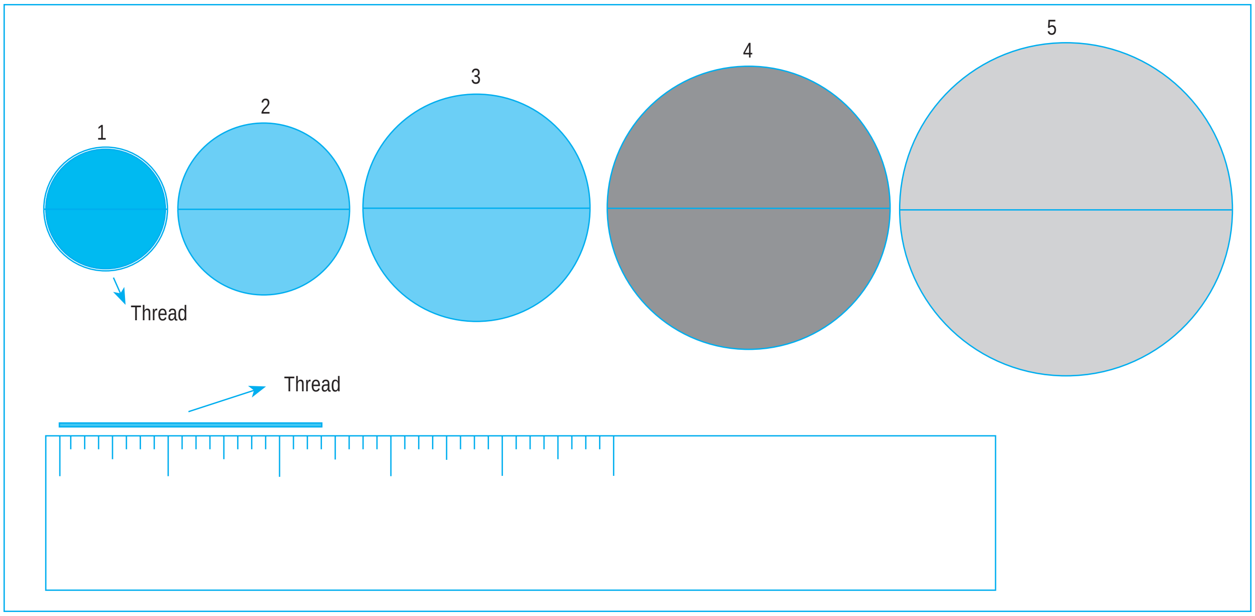
3. Now keep the thread at the circumference of circle 1 carefully so that it exactly covers the circumference. Measure this length of thread with ruler and note it down in the column of circle 1 of the above table.

4. Repeat this process for all the circles.

Result

We observe that the ratio of circumference to that of diameter of every circle *i.e.* $\frac{\text{circumference}}{\text{diameter}}$ is the same (constant). This ratio is the number π.

Thus we can say that circumference and diameter of a circle are in direct variation.



Example 1. Observe the following tables and find if x and y are directly proportional:

(*i*)

x	20	17	14	11	8	5	2
y	40	34	28	22	16	10	4

(*ii*)

x	6	10	14	18	22	26	30
y	4	8	12	16	20	24	28

Solution.

(*i*) $\frac{20}{40} = \frac{1}{2}, \frac{17}{34} = \frac{1}{2}, \frac{14}{28} = \frac{1}{2}, \frac{11}{22} = \frac{1}{2}, \frac{8}{16} = \frac{1}{2}, \frac{5}{10} = \frac{1}{2}, \frac{2}{4} = \frac{1}{2}$

Here, $\frac{x}{y} = \frac{20}{40} = \frac{17}{34} = \frac{14}{28} = \frac{11}{22} = \frac{8}{16} = \frac{5}{10} = \frac{2}{4} = \frac{1}{2} \Rightarrow \frac{x}{y} = \frac{1}{2}$ (constant)

$\therefore$ x and y are directly proportional.

(*ii*) $\frac{6}{4} = \frac{3}{2}, \frac{10}{8} = \frac{5}{4}, \frac{14}{12} = \frac{7}{6}, \frac{18}{16} = \frac{9}{8}, \frac{22}{20} = \frac{11}{10}, \frac{26}{24} = \frac{13}{12}, \frac{30}{28} = \frac{15}{14}$

Here, $\frac{6}{4} \neq \frac{10}{8}, \frac{10}{8} \neq \frac{14}{12}, \frac{14}{12} \neq \frac{9}{8}, \ldots$

$\Rightarrow \frac{x}{y}$ is not constant.

$\therefore$ x and y are not directly proportional.

Example 2. The cost of 5 metres of a particular quality of cloth is ₹ 210. Tabulate the cost of 2, 4, 10 and 15 metres of cloth of the same type.

Solution. Suppose length of cloth be x metres and its cost be ₹ y. Then

x	2	4	5	10	15
y	y_2	y_3	210	y_4	y_5

As the length of cloth increases or decreases, cost of cloth also increases or decreases in the same ratio. Hence, it is a case of direct variation.

So we have $\frac{x}{y}$ = constant

$$\Rightarrow \quad \frac{2}{y_2} = \frac{4}{y_3} = \frac{5}{210} = \frac{10}{y_4} = \frac{15}{y_5}.$$

Taking $\frac{2}{y_2} = \frac{5}{210} \Rightarrow 5y_2 = 2 \times 210 \Rightarrow y_2 = \frac{2 \times 210}{5} = 84.$

Taking $\frac{4}{y_3} = \frac{5}{210} \Rightarrow 5y_3 = 4 \times 210 \Rightarrow y_3 = \frac{4 \times 210}{5} = 168.$

Taking $\frac{10}{y_4} = \frac{5}{210} \Rightarrow 5y_4 = 10 \times 210 \Rightarrow y_4 = \frac{5 \times 210}{5} = 420.$

Taking $\frac{15}{y_5} = \frac{5}{210} \Rightarrow 5y_5 = 15 \times 210 \Rightarrow y_5 = \frac{15 \times 210}{5} = 630.$

Hence

x	2	4	5	10	15
y	84	168	210	420	630

Example 3. If 7 kg sugar costs ₹ 115.50, what is the cost of 12 kg sugar?

Solution. Let the cost of 12 kg sugar be ₹ x.

Note that more the mass of sugar, more would be the cost of sugar.

Hence, it is a case of direct variation.

Mass of sugar (in kg)	7	12
Cost of sugar (in ₹)	115.50	x

$\therefore \quad \frac{7}{115.50} = \frac{12}{x} \Rightarrow 7x = 12 \times 115.50$

$\Rightarrow \quad x = \frac{12 \times 115.50}{7} \Rightarrow x = 198.$

$\therefore$ Cost of 12 kg sugar = ₹ 198.

Example 4. An electric pole, 14 metres high, casts a shadow of 10 metres. Find the height of a tree that casts a shadow of 15 metres under similar conditions.

Solution. Let the height of tree be x metres. Note that more the height of an object, the more would be the length of its shadow.

Height (in metres)	14	x
Length of shadow (in metres)	10	15

Hence, it is a case of direct variation.

$\therefore \quad \frac{14}{10} = \frac{x}{15} \Rightarrow 10x = 14 \times 15 \Rightarrow x = \frac{14 \times 15}{10} = 21.$

$\therefore$ Height of the tree is 21 metres.

Example 5. A machine in a soft drink factory fills 840 bottles in 6 hours. How many bottles will it fill in 5 hours?

Solution. Let the machine fill x number of bottles in 5 hours. Note that in less time, less number of bottles will be filled by the machine. Hence, it is a case of direct variation.

Time (in hours)	6	5
Number of bottles	840	x

$\therefore \quad \frac{6}{840} = \frac{5}{x} \Rightarrow 6x = 840 \times 5 \Rightarrow x = \frac{840 \times 5}{6} \Rightarrow x = 700.$

Hence, the machine will fill 700 bottles in 5 hours.

Example 6. The scale of a map is given as 1 : 30000000. Two cities are 4 cm apart on the map. Find the actual distance between them.

Solution. Let the actual distance between two cities be x cm. Note that more the map distance, the more would be the actual distance.

Map distance (in cm)	1	4
Actual distance (in cm)	30000000	x

Hence, it is a case of direct variation.

$\therefore \frac{1}{30000000} = \frac{4}{x} \Rightarrow x = 4 \times 30000000 = 120000000$

$\therefore$ Actual distance between the cities = 120000000 cm

$$= \frac{120000000}{100 \times 1000} \text{ km} \qquad \left(\because 1\text{cm} = \frac{1}{100 \times 1000}\text{km}\right)$$

$$= 1200 \text{ km}$$

Example 7. If 3 men or 4 women can earn ₹ 480 in a day, find how much will 6 men and 11 women earn in a day?

Solution. Earning of 3 men = earning of 4 women

$\Rightarrow$ Earning of 1 man = earning of $\frac{4}{3}$ women

$\Rightarrow$ Earning of 6 men = earning of $\left(\frac{4}{3} \times 6\right)$ women

= earning of 8 women.

$\Rightarrow$ Earning of 6 men and 11 women = earning of (8 + 11) women

= earning of 19 women.

Let 6 men and 11 women will earn ₹ x in a day.

Note that more women will earn more per day.

Hence, it is a case of direct variation.

Number of women	4	19
Earning per day (in ₹)	480	x

$\therefore \quad \frac{4}{480} = \frac{19}{x} \Rightarrow 4x = 480 \times 19 \Rightarrow x = \frac{480 \times 19}{4} = 2280.$

Hence, 6 men and 11 women will earn ₹ 2280 in a day.

Exercise 8.1

1. Observe the following tables and find if x and y are directly proportional:

(*i*)

x	5	8	12	15	18	20
y	15	24	36	60	72	100

(*ii*)

x	3	5	7	9	10
y	9	15	21	27	30

2. If x and y are in direct variation, complete the following tables:

(*i*)

x	3	5	…	…	10
y	45	…	90	120	….

(*ii*)

x	4	8	…	20	28
y	7	…	21	…	…

3. If 8 metres cloth costs ₹ 250, find the cost of 5.8 metres of the same cloth.

4. If one dozen pencils cost ₹ 21, find the cost of one score pencils.

5. If 18 dolls costs ₹ 450, how many dolls can be purchased for ₹ 325?

6. If a labourer earns ₹ 672 per week, how much will he earn in 18 days?

7. If 175 dollars cost ₹ 7350, how many dollars can be purchased in ₹ 24024?

8. If a car travels 67.5 km in 4.5 litres of petrol, how many kilometres will it travel in 26.4 litres of petrol?

9. If the weight of 12 sheets of thick paper is 40 grams, how many sheets of the same paper would weight $2\frac{1}{2}$ kilograms?

10. A train is moving at a uniform speed of 75 km/hour.

(*i*) How far will it travel in 20 minutes?

(*ii*) Find the time required to cover a distance of 250 km.

11. A photograph of a bacteria enlarged 50000 times attains a length of 5 cm. What is the actual length of the bacteria? If the photograph is enlarged 20000 times only, what would be its enlarged length?

12. In a model of a ship, the mast (flag staff) is 9 cm high, while the mast of the actual ship is 12 m high. If the length of the ship is 28 m, how long is the model of the ship?

13. Suppose 2 kg of sugar contains 9×10^6 crystals. How many sugar crystals are there in (*i*) 5 kg of sugar? (*ii*) 1.2 kg of sugar?

14. Rashmi has a road map with a scale of 1cm representing 18 km. She drives on a road for 72 km. What would be her distance covered in the map?

15. A 5 m 60 cm high vertical pole casts a shadow 3 m 20 cm long. Find at the same time
(*i*) the length of the shadow cast by another pole 10 m 50 cm high
(*ii*) the height of pole which casts a shadow 5 m long.

16. A loaded truck travels 14 km in 25 minutes. If the speed remains same, how far can it travel in 5 hours?

17. If 5 men or 7 women can earn ₹ 525 per day, how much would 10 men and 13 women will earn per day?

INVERSE VARIATION

Two quantities are said to ***vary inversely*** *if the increase (or decrease) in one quantity causes the decrease (or increase) in the other quantity*.

Foe example:

(*i*) The time taken to finish a work varies inversely to the number of men at work.

More men at work, less time taken to finish the work.

Less men at work, more time taken to finish the work.

(*ii*) The speed of a moving object varies inversely to the time taken to cover a certain distance.

More speed, less time taken to cover the same distance.

Less speed, more time taken to cover the same distance.

Let us consider the following:

Suppose distance between Reeta's house and her office is 40 km. She take 4 hours to reach the office if she travels at a speed of 10 km/hr. If she travels at a faster speed of 20 km/hr, she takes 2 hours to reach. Similarly, we can find the time taken by her to reach the office if she travels at a speed of 40 km/hr or 80 km/hr.

Study the following table:

Speed in km/hour (x)	10	20	40	80
Time taken in hour (y)	4	2	1	$\frac{1}{2}$

We find that as x increases, y decreases and xy is constant. In this case

$$xy = 10 \times 4 = 20 \times 2 = 40 \times 1 = 80 \times \frac{1}{2} = 40 \text{ (constant)}$$

i.e. $xy = k$(constant), where k is called **constant of variation**.

So we can say that:

Two quantities x and y are said to be in inverse variation if xy is constant.

Example 1. Observe the following tables and find which pair of variables (here x and y) are in inverse variation:

(*i*)

x	50	40	30	20
y	5	6	7	8

(*ii*)

x	100	200	300	400
y	60	30	20	15

Solution.

(*i*) $50 \times 5 = 250$, $40 \times 6 = 240$, $30 \times 7 = 210$, $20 \times 8 = 160$