10 Triangle and its Properties

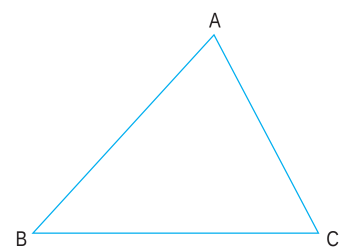
INTRODUCTION

In class VI, you have already learnt about triangles and the types of triangles on the basis of sides and also on the basis of angles. In this chapter, we shall review, revise and strengthen these. We shall also learn the following properties of a triangle:

- Exterior angle of a triangle and its property
- Angle sum property of a triangle
- Angle property of special triangles
- Sum of lengths of two sides of a triangle
- Pythagoras property of a right angled triangle

TRIANGLE

A **triangle** *is a simple closed curved made of three line segments.*



In the adjoining figure, ABC is a triangle. Usually, triangle ABC is written as ΔABC. It has three sides – $\overline{AB}$, $\overline{BC}$, $\overline{CA}$. It has three angles – $\angle BAC$, $\angle ABC$, $\angle ACB$. It has three vertices – A, B, C.

Thus, a triangle has three sides and three angles, and all the three sides and all the three angles are called **six elements** of the triangle ABC. In the above figure, look at the vertex A. It is the point of intersection of the sides AB and AC, BC is the remaining side. We say that vertex A and side BC are opposite to each other. Also $\angle A$ and side BC are opposite to each other.

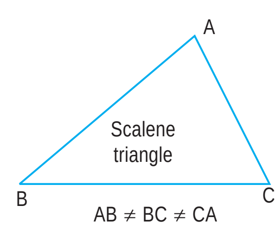
Similarly, vertex B and side CA are opposite to each other; $\angle B$ and side CA are opposite to each other. Same can be said about vertex C, $\angle C$ and side AB.

Types of triangles on the basis of sides

(*i*) **Scalene triangle**

If all the sides of a triangle are unequal, it is called a ***scalene triangle***.

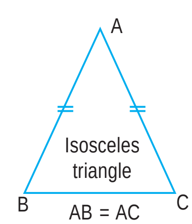
In the adjoining diagram, $AB \neq BC \neq CA$, so ΔABC is a scalene triangle.



(*ii*) **Isosceles triangle**

If any two sides of a triangle are equal, it is called an ***isosceles triangle***.

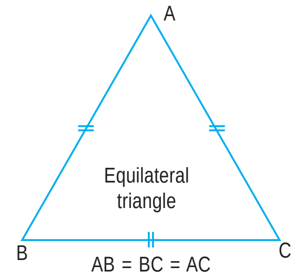
In the adjoining diagram, $AB = AC$, so ΔABC is an isosceles triangle. Usually, equal sides are indicated by putting marks on each of them.



(*iii*) **Equilateral triangle**

*If all the three sides of a triangle are equal, it is called an **equilateral triangle**.*

In the adjoining diagram, AB = BC = AC, so ΔABC is an equilateral triangle.

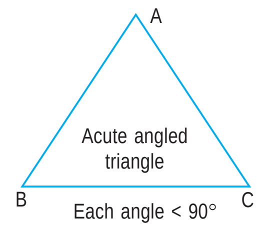


Types of triangles on the basis of angles

(*i*) **Acute angled triangle**

*If all the three angles of a triangle are acute (less than 90°), it is called an **acute angled triangle**.*

In the adjoining diagram, each angle is less than 90°, so ΔABC is an acute angled triangle.

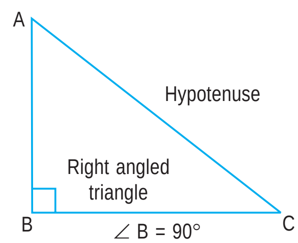


(*ii*) **Right angled triangle**

*If one angle of a triangle is right angle (= 90°), it is called a **right angled triangle**.*

In a right angled triangle, the side opposite to right angle is called **hypotenuse**.

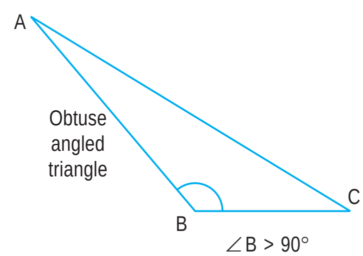
In the adjoining diagram, ∠B = 90°, so ΔABC is a right angled triangle and side AC is the hypotenuse.



(*iii*) **Obtuse angled triangle**

*If one angle of a triangle is obtuse (greater than 90°), it is called an **obtuse angled triangle**.*

In the adjoining diagram, ∠B is obtuse (greater than 90°), so ΔABC is an obtuse angled triangle.

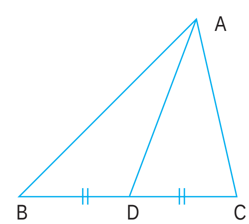


Medians of a triangle

*The line segment joining a vertex of a triangle to the mid-point of the opposite side is called a **median** of the triangle.*

In adjoining diagram, AD is median from A to the side BC. A median lies wholly in the interior of a triangle.

A triangle has three medians.

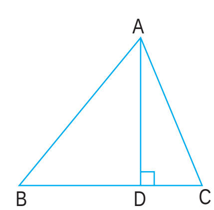


Altitudes of a triangle

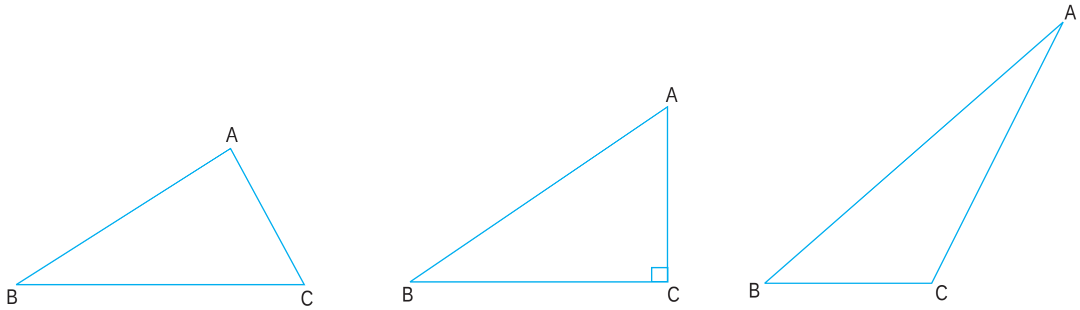
*The perpendicular line segment from a vertex of a triangle to the opposite side is called an **altitude** of the triangle.*

In adjoining diagram, AD is an altitude from A to the side BC.

A triangle has three altitudes.



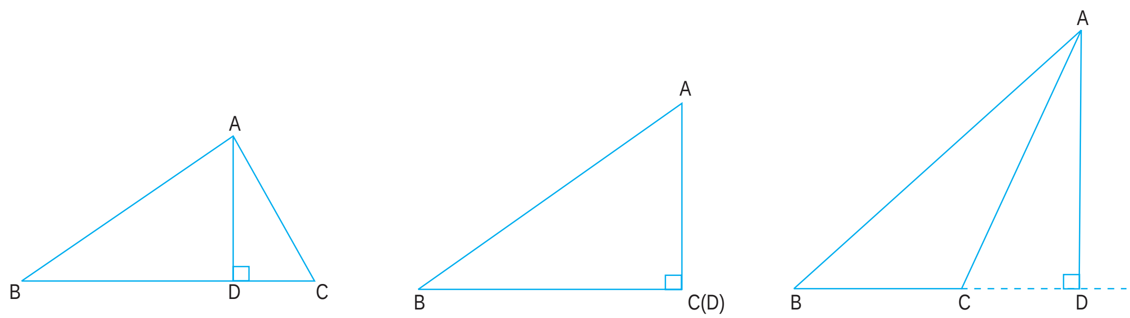
Example 1. Draw rough sketches of altitudes from vertex A to side BC for the following triangles:



(*i*) (*ii*) (*iii*)

Also check whether the altitude lies in the interior or the exterior or it is a side itself of ΔABC.

Solution. Rough sketches of altitude AD from vertex A to side BC of ΔABC in each figure is shown below:



(*i*) (*ii*) (*iii*)

We observe that:

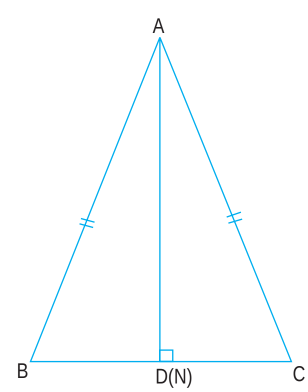
In fig. (*i*), the altitude AD lies wholly in the interior of ΔABC. In fact, all the three altitudes will lie wholly in the interior of an acute angled triangle.

In fig. (*ii*), the altitude AD coincides with the side AC *i.e.* it is the side AC itself. Further note that the altitude from vertex B to side AC is the side BC itself. In fact, in a right angled triangle, two sides containing the right angle are themselves altitudes and the third altitude will lie wholly in the triangle.

In fig. (*iii*), the altitude AD lies completely in the exterior of ΔABC. In fact, in an obtuse angled triangle, two altitudes lies in the exterior and one altitude will lie in the interior of the triangle.

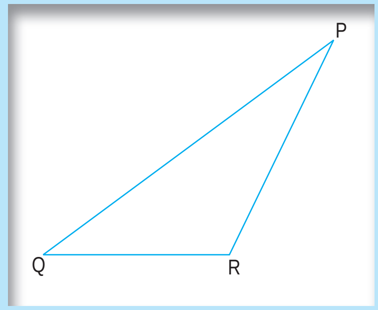
Example 2. Verify by drawing a diagram whether a median and an altitude of an isosceles triangle can be same.

Solution. Let ABC be an isosceles triangle with AB = AC. Find the mid-point D of side BC (by locating the perpendicular bisector of segment BC by paper folding or by construction). Then the line segment joining points A and D is a median of ΔABC.



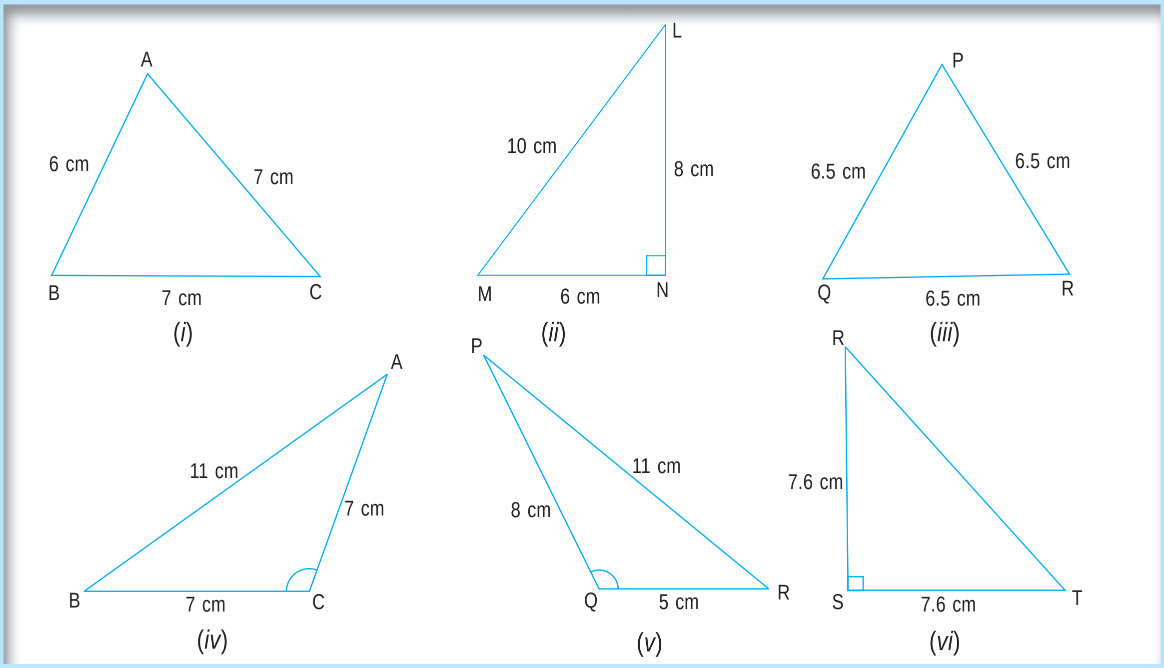
From A, draw AN perpendicular to BC. Then AN is an altitude of ΔABC. We note that the point N coincides with point D *i.e.* AD and AN are same. Hence, a median and an altitude of an isosceles triangle are same.

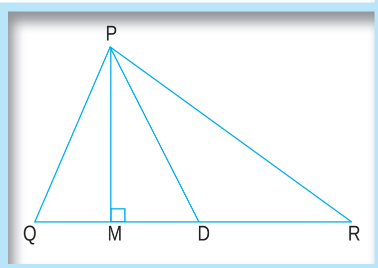
Exercise 10.1



1. In the adjoining figure:
   (*i*) Name the vertex opposite to side PQ.
   (*ii*) Name the side opposite to vertex Q.
   (*iii*) Name the angle opposite to side QR.
   (*iv*) Name the side opposite to ∠R.

2. Look at the figures given below and classify each of the triangle according to its
   (*a*) sides (*b*) angles
   (You may judge the nature of the angle by observation):





3. In the adjoining ΔPQR, if D is the mid-point of $\overline{QR}$, then
   (*i*) $\overline{PM}$ is ..... (*ii*) $\overline{PD}$ is .....
   Is QM = MR?

4. Will an altitude always lie in the interior of a triangle? If no, draw a rought sketch to show such a case.

5. Can you think of a triangle in which two altitudes of the triangle are its sides? If yes, draw a rough sketch to show such a case.

6. Draw rough sketches for the following:
   (*i*) In Δ ABC, BE is a median of the triangle.
   (*ii*) In ΔPQR, PQ and PR are altitudes of the triangle.
   (*iii*) In ΔXYZ, YL is an altitude in the exterior of the triangle.

7. Take an equilateral triangle and draw its medians and altitudes and check that the medians and altitudes are same.