7 Ratio and Proportion

INTRODUCTION

You learnt the basic ideas of ratio and proportion in the previous class.

A politician inspecting a school asked the students — 'There are 30 students in this class and my monthly salary is ₹ 30000. So, what is my age?' All students were puzzled how to solve this problem. Then one student said — 'Sir, your age is 40 years'. Now the politician was puzzled. 'How did you find out my correct age?' The student replied — 'Sir, my elder brother is half mad and he is 20 years old.'

Well, that is an example of direct proportion. In this chapter, we shall review and strengthen the ideas learnt in previous class and will add a few tougher problems.

We shall also review and strengthen the unitary method.

RATIO

A **ratio** is a comparison of measures (or magnitudes) of two or more quantities of the same kind by division.

If a and b are two quantities of the same kind (in same units), then the fraction $\frac{a}{b}$ *is called the* ***ratio*** *of a to b*. It is written as $a : b$ (read as '*a*' is to '*b*').

Thus, the ratio of a to $b = \frac{a}{b}$ or $a : b$.

The quantities a and b are called the **terms** of the ratio, a is called the **first term** (or **antecedent**) and b is called the **second term** (or **consequent**).

For example:

(*i*) The ratio of ₹ 9 to ₹ 16 = $\frac{9}{16}$ or 9 : 16.

(*ii*) The ratio of 150 cm to 100 cm = $\frac{150}{100} = \frac{3}{2}$ or 3 : 2.

(*iii*) The ratio of 8 marbles to 12 marbles = $\frac{8}{12} = \frac{2}{3}$ or 2 : 3.

Remarks

- Since ratio is a fraction, both of its terms (numerator and denominator) can be multiplied or divided by the same (non-zero) number.

For example:

(*i*) $3 : 5 = \frac{3}{5} = \frac{3 \times 4}{5 \times 4} = \frac{12}{20} \Rightarrow 3 : 5 = 12 : 20.$

(*ii*) $16 : 24 = \frac{16}{24} = \frac{16 \div 8}{24 \div 8} = \frac{2}{3} \Rightarrow 16 : 24 = 2 : 3.$

Usually, a ratio is expressed in lowest terms (or simplest form).

- *The order of the terms in a ratio is important.*
  The ratio 3 : 5 is different from the ratio 5 : 3.
- *Ratio exists only between two quantities of the same kind.*

For example:

(*i*) There is no ratio between number of students in a class and the salary of a politician.

(*ii*) There is no ratio between the weight of one child and the age of another child.

- *Quantities to be compared (by division) must be in the same units.*

For example:

(*i*) Ratio between 150 g and 2 kg = ratio between 150 g and 2000 g

$$= \frac{150}{2000} = \frac{3}{40} = 3 : 40.$$

(*ii*) Ratio between 25 minutes and 45 seconds

= ratio between (25 × 60) sec and 45 sec

$$= \frac{1500}{45} = \frac{100}{3} = 100 : 3.$$

- *Ratio is a number, so it has no units.*
- *If the terms of a ratio are in fractions, convert them to natural numbers by multiplying each term by the L.C.M. of their denominators.*

Equivalent ratios

Two ratios are called equivalent if the fractions corresponding to them are equivalent.

Thus, the ratio 3 : 5 is equivalent to the ratio 9 : 15 or 21 : 35.

Ratio $a : b : c$

Three quantities of the same kind (in same units) are said to be in the ratio a : b : c if the quantities are ak, bk and ck respectively, where k is any positive number.

Similarly, four quantities of the same kind (in same units) are said to be in the ratio $a : b : c : d$ if the quantities are ak, bk, ck and dk respectively, where k is any positive number.

Example 1. Simplify the following ratios:

(*i*) $2\frac{2}{3} : 1\frac{1}{15}$ (*ii*) $\frac{1}{3} : \frac{1}{6} : \frac{1}{8}$.

Solution.

(*i*) Given ratio $= 2\frac{2}{3} : 1\frac{1}{15} = \frac{8}{3} : \frac{16}{15} = \dfrac{\frac{8}{3}}{\frac{16}{15}}$

$$= \frac{8}{3} \times \frac{15}{16} = \frac{5}{2} = 5 : 2.$$

(*ii*) Given ratio $= \frac{1}{3} : \frac{1}{6} : \frac{1}{8}$

L.C.M. of 3, 6 and 8 = 24

$$= \frac{1}{3} \times 24 : \frac{1}{6} \times 24 : \frac{1}{8} \times 24$$

= 8 : 4 : 3.

Example 2. A line segment of 1 metre length is divided into two parts such that the first part is $\frac{2}{3}$ of second part. Find the lengths of two parts in centimetres.

Solution. Length of line segment = 1 metre = 100 centimetres.

Ratio of lengths of two parts = $\frac{2}{3} : 1 = 2 : 3$.

Sum of the terms of the ratio = 2 + 3 = 5.

$\therefore$ Length of first part = $\frac{2}{5}$ of 100 cm = $\left(\frac{2}{5}\times 100\right)$ cm = 40 cm,

length of second part = $\frac{3}{5}$ of 100 cm = $\left(\frac{3}{5}\times 100\right)$ cm = 60 cm.

Example 3. Divide ₹ 260 among three children in the ratio $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$.

Solution. First we simplify the given ratio.

L.C.M. of 2, 3 and 4 = 12

$$\text{Given ratio} = \frac{1}{2} : \frac{1}{3} : \frac{1}{4}$$
$$= \frac{1}{2} \times 12 : \frac{1}{3} \times 12 : \frac{1}{4} \times 12$$
$$= 6 : 4 : 3.$$

Thus, we are to divide ₹ 260 in the ratio 6 : 4 : 3.

Sum of the terms of the ratio = 6 + 4 + 3 = 13.

$\therefore$ Share of first child = $\frac{6}{13}$ of ₹ 260 = ₹ $\left(\frac{6}{13}\times 260\right)$ = ₹ 120,

share of second child = $\frac{4}{13}$ of ₹ 260 = ₹ $\left(\frac{4}{13}\times 260\right)$ = ₹ 80,

share of third child = $\frac{3}{13}$ of ₹ 260 = ₹ $\left(\frac{3}{13}\times 260\right)$ = ₹ 60.

Example 4. A certain sum of money has been divided into two parts in the ratio 5 : 8. If the first part is ₹ 250, find the total amount.

Solution. Let the total amount be ₹ x.

The amount has been divided into two parts in the ratio 5 : 8.

Sum of the terms of the ratio = 5 + 8 = 13.

Then first part = $\frac{5}{13}$ of the total amount.

According to given condition, $\frac{5}{13}$ of ₹ x = ₹ 250

$\Rightarrow \frac{5}{13} \times x = 250 \Rightarrow x = \frac{250\times 13}{5} = 50 \times 13 = 650.$

Hence, the total amount = ₹ 650.

Alternative method

The amount has been divided into two parts in the ratio 5 : 8.

Let the two parts be ₹ $5x$ and ₹ $8x$.

Then the total amount = ₹ $5x$ + ₹ $8x$ = ₹ $13x$.

According to given condition, $5x = 250 \Rightarrow x = 50$.

$\therefore$ Total amount = ₹ $13x$ = ₹ (13×50) = ₹ 650.

Example 5. A natural number has been divided into two parts in the ratio 5 : 9. If the difference of these parts is 24, find the number and the two parts.

Solution. As the natural number has been divided into two parts in the ratio 5 : 9, let the two parts be $5x$ and $9x$.

Then the difference of these parts = $9x - 5x = 4x$.

According to given, $4x = 24$

$\Rightarrow$ $x = 6$.

$\therefore$ The required natural number = $5x + 9x = 14x = 14 \times 6 = 84$.

The two parts are $5x$ and $9x$ *i.e.* 5×6 and 9×6 *i.e.* 30 and 54.

Hence, the two parts are 30 and 54.

Example 6. Which ratio is greater — 11 : 21 or 19 : 28 ?

Solution. $11 : 21 = \frac{11}{21}$ and $19 : 28 = \frac{19}{28}$.

L.C.M. of 21 and 28 is 84.

Convert into equivalent like fractions

$$\frac{11}{21} = \frac{11 \times 4}{21 \times 4} = \frac{44}{84} \text{ and } \frac{19}{28} = \frac{19 \times 3}{28 \times 3} = \frac{57}{84}.$$

As $57 > 44$, $\frac{57}{84} > \frac{44}{84} \Rightarrow \frac{19}{28} > \frac{11}{21}$.

Hence, 19 : 28 is the greater ratio.

Exercise 7.1

1. Express the following ratios in simplest form:

(*i*) $\frac{1}{6} : \frac{1}{9}$ (*ii*) $4\frac{1}{2} : 1\frac{1}{8}$ (*iii*) $\frac{1}{5} : \frac{1}{10} : \frac{1}{15}$.

2. Find the ratio of each of the following in simplest form:

(*i*) ₹ 5 to 50 paise (*ii*) 3 km to 300 m

(*iii*) 9 m to 27 cm (*iv*) 15 kg to 210 g

(*v*) 25 minutes to 1.5 hours (*vi*) 30 days to 36 hours.

3. In a class of 55 students, the number of girls is 25. Find the ratio of number of

(*i*) girls to the total number of students

(*ii*) boys to the total number of students

(*iii*) boys to the number of girls.

4. Out of daily income of ₹ 120, a labourer spends ₹ 90 on food and shelter and saves the rest. Find the ratio of his

(*i*) spending to income (*ii*) saving to income (*iii*) saving to spending.

5. 5 grams of an alloy contains $3\frac{3}{4}$ grams copper and the rest is nickel. Find the ratio by weight of nickel to copper.

6. A pole of height 3 metres is struck by a speeding car and breaks into two pieces such that the first piece is $\frac{1}{2}$ of the second. Find the length of both pieces.

7. Divide 25 toffees between two children in the ratio $\frac{1}{2} : \frac{1}{3}$.

8. Heights of Anshul and Dhruv are 1.04 m and 78 cm respectively. Divide 35 sweets between them in the ratio of their heights.

[Hint. Height of Anshul : Height of Dhruv = $\frac{104}{78} = \frac{4}{3}$ = 4 : 3.]

9. ₹ 180 are to be divided among three children in the ratio $\frac{1}{3} : \frac{1}{4} : \frac{1}{6}$. Find the share of each child.

10. Amit, Kunal and Suresh started a business and invested ₹ 15000, ₹ 10000 and ₹ 20000 respectively. In the first month, they made a profit of ₹ 7542 and they decided to share the profit in the ratio of their investments. Find the share of profit that each of them will get.

11. Two numbers are in the ratio 4 : 7 and their difference is 18. Find the numbers.

12. A natural number has been divided into two parts in the ratio 7 : 11. If the difference of two parts is 20, find the number and the two parts.

13. A certain sum of money has been divided into two parts in the ratio 9 : 13. If the second part is ₹ 260, find the total amount.

14. Are the ratios 1 : 2 and 2 : 3 equivalent?

15. Following is the performance of a cricket team in the matches it played:

Year	Wins	Losses
Last year	8	2
This year	4	2

In which year was the record better?

16. Which ratio is greater :

(*i*) 5 : 6 or 6 : 7 (*ii*) 13 : 24 or 17 : 32.

PROPORTION

An equality of two ratios is called a **proportion.**

Four quantities a, b, c and d are said to be in ***proportion*** *if a : b = c : d i.e. if*

$\frac{a}{b} = \frac{c}{d}$ *i.e. if ad = bc.*

This proportion is also written as *a* : *b* : : *c* : *d*.

For example : 2, 3, 4, 6 are in proportion since $\frac{2}{3} = \frac{4}{6}$.

Four quantities *a*, *b*, *c* and *d* are called **terms** of the proportion *a* : *b* : : *c* : *d* and *a*, *b*, *c* and *d* are called its first, second, third and fourth terms respectively.

First and fourth terms are called **extremes** (or **extreme terms**).

Second and third terms are called **means** (or **middle terms**).

If *a* : *b* = *c* : *d*, then *d* is called the **fourth proportional**.

If *a*, *b*, *c* and *d* are in proportion, then $\frac{a}{b} = \frac{c}{d}$ *i.e. ad = bc i.e.*

product of extreme terms = product of middle terms.

Thus, a, b, c and *d are in proportion if*

product of extremes = product of means.

This is known as **cross product rule**.

Hence, if *ad* ≠ *bc*, then *a*, *b*, *c* and *d* are not in proportion.