13 Perimeter and Area

INTRODUCTION

In class VI, we have already learnt about perimeters of closed plane figures and areas of squares and rectangles. Perimeter of a closed plane figure is the length of its boundary and area is the measure of the part of plane or region enclosed by its boundary.

In this chapter, we shall review, revise and strengthen these and will learn about perimeter and area of some more closed plane figures.

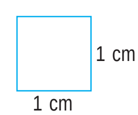
SQUARES AND RECTANGLES

The ***perimeter*** *of a closed plane figure is the length of its boundary*. The unit of measurement of the perimeter is the same as that of length.

The ***area*** *of a closed plane figure is the measure of the region (part of plane) enclosed by its boundary*.

To measure the area of a closed plane figure, we have to compare it with some unit of area. For convenience, the shape we choose for the unit of area is a unit square.

If the side of a square is 1cm, then the unit is called a square centimetre (written as cm^2) and if the side is 1 metre, then the unit is called a square metre (written as m^2) etc.



Perimeter and area of a square

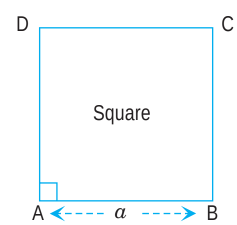
Perimeter of a square = 4 × length of a side.

Thus, $P = 4a$, where P = perimeter and a = length of a side of the square.

Area of a square = (length of a side)².

Thus, $A = a^2$, where A = area and

a = length of a side of the square.

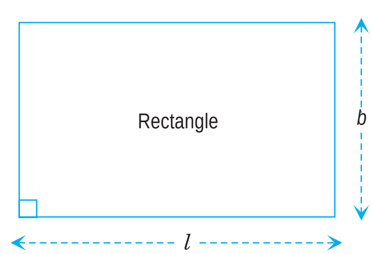


Perimeter and area of a rectangle

Perimeter of a rectangle = 2 (length + breadth).

Thus, $P = 2(l + b)$, where P = perimeter, l = length and b = breadth of rectangle.

$\therefore \quad l = \frac{P}{2} - b,\ b = \frac{P}{2} - l.$



Area of a rectangle = length × breadth.

Thus, $A = l \times b$, where A = area, l = length and b = breadth of rectangle.

$\therefore \quad l = \frac{A}{b}, \quad b = \frac{A}{l}.$

Note. The length and the breadth of a rectangle must be in the same units.

Conversion of units

Units of length	Units of area
1 cm = 10 mm	1 cm^2 = 10 mm × 10 mm = 100 mm^2
1 m = 100 cm	1 m^2 = 100 cm × 100 cm = 10000 cm^2
1 km = 1000 m	1 km^2 = 1000 m × 1000 m = 1000000 m^2
	1 hectare = 100 m × 100 m = 10000 m^2

In short, hectare is written as 'ha'.

Example 1. Find the area of a rectangle whose length and breadth are 2.7 m and 85 cm.

Solution. The length and the breadth must be expressed in the same units.

Length = 2.7 m and breadth = 85 cm = $\frac{85}{100}$ m = 0.85 m.

$\therefore$ Area of rectangle = length × breadth

= (2.7×0.85) m^2 = 2.295 m^2.

Alternatively

Length = 2.7 m = (2.7×100) cm = 270 cm and breadth = 85 cm.

$\therefore$ Area of the rectangle = (270×85) cm^2 = 22950 cm^2.

Example 2. The perimeter of a square is 26 cm. Find

(*i*) the length of a side (*ii*) area of the square.

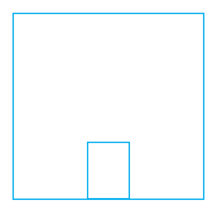
Solution.

(*i*) Perimeter of a square = 4 (length of a side)

$\Rightarrow$ length of a side = $\frac{\text{perimeter of the square}}{4} = \frac{26}{4}$ cm = 6.5 cm.

(*ii*) Area of the square = $(\text{length of a side})^2 = (6.5 \text{ cm})^2 = 42.25$ cm^2.

Example 3. A door-frame of dimensions 3 m × 2 m is fixed on the wall of dimensions 10 m × 10 m. Find the total labour charges for painting the wall if the labour charges for painting 1m^2 of the wall is ₹ 2.50



Solution. Painting of the wall has to be done excluding the area of the door.

Area of wall (including door) = 10 m × 10 m = 100 m^2,

area of door = 3 m × 2 m = 6 m^2

$\therefore$ Area of wall to be painted = total area of wall – area of door

= (100 – 6) m^2 = 94 m^2

$\therefore$ Total labour charges for painting the wall = ₹ (2.50×94) = ₹ 235.

Example 4. The area of a rectangular sheet is 500 cm^2. If the length of the sheet is 25 cm, what is its breadth? Also find the perimeter of the rectangular sheet.

Solution. Let the breadth of the rectangular sheet be b cm.

$\therefore$ Area of sheet = length × breadth = 25 cm × b cm = 500 cm^2 (given)

$\Rightarrow 25b = 500 \Rightarrow b = \frac{500}{25} = 20.$

$\therefore$ The breadth of rectangular sheet = 20 cm.

Perimeter of rectangular sheet = $2(l + b) = 2(25 + 20)$ cm = 90 cm.

Example 5. A wire is in the shape of a square of side 10 cm. If the wire is rebent into a rectangle of length 12 cm, find its breadth. Which shape encloses more area and by how much?

Solution. Length of wire = perimeter of square = 4 × side

= 4 × 10 cm = 40 cm

As the wire is rebent into the shape of a rectangle,

perimeter of rectangle = length of wire = 40 cm.

Let the breadth of the rectangle be b cm.

Perimeter of rectangle = 2 (length + breadth)

$\Rightarrow$ 40 cm = 2 (12 + b) cm $\Rightarrow$ 40 = 2 (12 + b)

$\Rightarrow$ 20 = 12 + b $\Rightarrow$ b = 20 – 12 $\Rightarrow$ b = 8.

$\therefore$ The breadth of rectangle = 8 cm

Area of square = 10 cm × 10 cm = 100 cm^2,

area of rectangle = 12 cm × 8 cm = 96 cm^2.

$\therefore$ Area of square – area of rectangle = (100 – 96) cm^2 = 4 cm^2.

Hence, square has more area by 4 cm^2.

Example 6. The area of a square and a rectangle are equal. If the side of the square is 40 cm and the breadth of the rectangle is 25 cm, find the length of the rectangle. Also, find the perimeter of the rectangle.

Solution. Area of a square = $(\text{side})^2$ = 40 cm × 40 cm = 1600 cm^2.

As the area of rectangle = area of square

$\Rightarrow$ area of rectangle = 1600 cm^2

Let the length of the rectangle by l cm.

Area of rectangle = length × breadth

$\Rightarrow$ l cm × 25 cm = 1600 cm^2 $\Rightarrow$ 25 l = 1600

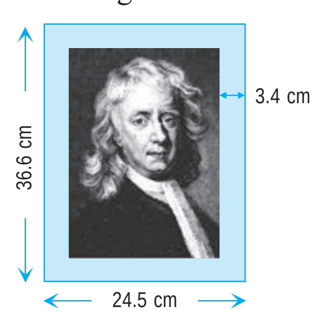
$\Rightarrow$ $l = \frac{1600}{25}$ $\Rightarrow$ l = 64.

$\therefore$ The length of rectangle = 64 cm.

Perimeter of rectangle = 2 (length + breadth)

= 2 (64 cm + 25 cm) = (2 × 89) cm = 178 cm.

Example 7. The rectangular frame of a picture is 24.5 cm wide and 36.6 cm high. The border is 3.4 cm wide. Calculate the perimeter of the picture.



Solution. Width of picture = 24.5 cm – (3.4 cm + 3.4 cm)

= 24.5 cm – 6.8 cm

= 17.7 cm

Height of picture = 36.6 cm – (3.4 cm + 3.4 cm)

= 36.6 cm – 6.8 cm

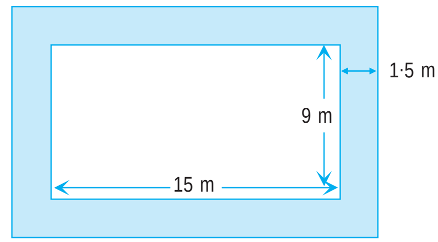
= 29.8 cm.

$\therefore$ Perimeter of picture = 2 (width + height)

$= 2\ (17.7 \text{ cm} + 29.8 \text{ cm})$

$= 2 \times 47.5 \text{ cm} = 95 \text{ cm}.$

Example 8. A rectangular lawn is 15 m long and 9 m wide. It has a path 1.5 m wide all around it. Find the area of the path.



Solution. In the adjoining figure, the shaded portion shows the path.

Area of the lawn $= (15 \times 9)\ \text{m}^2$

$= 135\ \text{m}^2.$

The length of the outer rectangle $= 15 \text{ m} + 1.5 \text{ m} + 1.5 \text{ m}$

$= 18 \text{ m}$

and the breadth of the outer rectangle $= 9 \text{ m} + 1.5 \text{ m} + 1.5 \text{ m} = 12 \text{ m}.$

$\therefore$ Total area of the lawn and the path $= (18 \times 12)\ \text{m}^2 = 216\ \text{m}^2.$

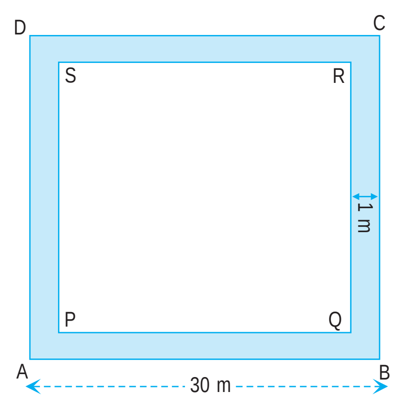
$\therefore$ The area of the path $= 216\ \text{m}^2 - 135\ \text{m}^2 = 81\ \text{m}^2.$

Example 9. A path 1 metre wide is built along the border and inside a square garden of side 30 m. Find:

(*i*) the area of the path.

(*ii*) the cost of planting grass in the remaining portion of the garden at the rate of ₹ 40 per m^2.

Solution.



(*i*) Let ABCD be a square garden of side 30 m. A path of 1 metre wide is built along the border inside.

In the adjoining figure, the shaded portion shows the path.

Note that PQRS is a square of side

$= 30 \text{ m} - 1 \text{ m} - 1 \text{ m} = 28 \text{ m}.$

Area of square ABCD $= (\text{side})^2$

$= 30 \text{ m} \times 30 \text{ m} = 900\ \text{m}^2,$

area of square PQRS $= (\text{side})^2 = 28 \text{ m} \times 28 \text{ m} = 784\ \text{m}^2.$

$\therefore$ Area of path $= 900\ \text{m}^2 - 784\ \text{m}^2 = 116\ \text{m}^2$

(*ii*) Area of the remaining portion of the garden = area of square PQRS

$= 784\ \text{m}^2.$

$\therefore$ Cost of planting grass in the remaining portion of garden

= ₹ (40×784) = ₹ 31360.

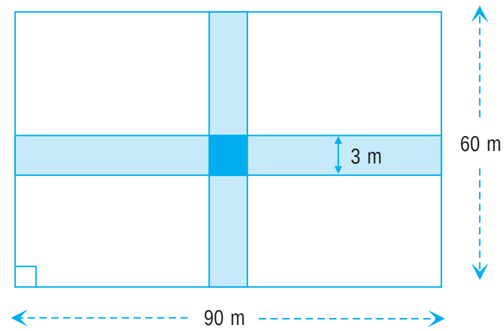
Example 10. Through a rectangular field of length 90 m and breadth 60 m, two cross roads are constructed which are parallel to the sides and cut each other at right angles through the centre of the field. If the width of each road is 3 m, find

(*i*) the area covered by the roads.

(*ii*) the cost of constructing the roads at the rate of ₹ 110 per m^2.

Solution.

(*i*) The area covered by the roads is shown shaded in the adjoining diagram.



Area of the shaded portion parallel to the length $= (90 \times 3)\ m^2 = 270\ m^2$.

Area of the shaded portion parallel to the breadth

$= (60 \times 3)\ m^2 = 180\ m^2$.

Area of the dark shaded portion

$= (3 \times 3)\ m^2 = 9\ m^2$.

$\therefore$ The area covered by the cross roads = area of shaded portion

$= 270\ m^2 + 180\ m^2 - 9\ m^2 = 441\ m^2$

Note that we subtract the area of the dark shaded portion because it has occurred twice – firstly when considering area parallel to length and secondly when considering area parallel to breadth.

(*ii*) Given, the cost of constructing the roads is at the rate of ₹ 110 per m^2.

$\therefore$ Total cost of constructing the roads = ₹ (110 × 441) = ₹ 48510.

Example 11. The perimeter of the floor of a rectangular room is 17 m and its length is 5 m. Find

(*i*) the breadth of the room.

(*ii*) the floor area of the room.

(*iii*) the cost of flooring the room with square tiles of side 25 cm at the rate of ₹ 850 per hundred tiles.

Solution.

(*i*) Let the breadth of the room be b metres.

Perimeter of the floor of the room = 2 (length + breadth).

$\therefore\ 17 = 2\,(5 + b) \Rightarrow 17 = 10 + 2b$

$\Rightarrow 2b = 17 - 10 = 7 \Rightarrow b = 3.5$

$\therefore$ The breadth of the room = 3.5 m.

(*ii*) Floor area of the room $= (5 \times 3.5)\ m^2 = 17.5\ m^2$.

(*iii*) Area of one tile $= (25 \times 25)\ cm^2 = 625\ cm^2$.

Area of the floor of the room $= 17.5\ m^2 = (17.5 \times 10000)\ cm^2$

$= 175000\ cm^2$

$\therefore$ The number of tiles required to cover the floor of the room

$$= \frac{\text{area of the floor}}{\text{area of one tile}} = \frac{175000}{625} = 280.$$

As the cost of 100 tiles = ₹ 850,

$\therefore$ the cost of 280 tiles = ₹ $\left(\frac{850}{100} \times 280\right)$ = ₹ 2380.

Hence, the cost of flooring the room with tiles = ₹ 2380.

Exercise 13.1

1. Complete the following table:

Side of square	*Perimeter of square*	*Area of square*
3.5 cm	...	...
7.8 m	...	...
...	44 cm	...

2. For a rectangle, complete the following table:

Length	*Breadth*	*Perimeter*	*Area*
7.5 cm	6 cm	...	...
4.4 m	2.8 m	...	...
2.3 m	90 cm	... m	... m^2

3. Anu wants to fence the garden in front of her house on three sides with lengths 12 m, 20 m and 12 m. Find the cost of fencing the garden at the rate of ₹ 150 per metre.

4. Find the area of a square park whose perimeter is 320 m.

5. Find the breadth of a rectangular plot of land, if its area is 440 m^2 and the length is 22 m. Also find its perimeter.

6. The perimeter of a rectangular sheet is 100 cm. If the length is 35 cm, find its breadth. Also find its area.

7. The perimeter of a rectangle is 130 cm. If the breadth of the rectangle is 30 cm, find its length. Also find the area of the rectangle.

8. The area of a rectangular plot is 340 m^2 and its breadth is 17 m. Find the cost of surrounding the plot with a fence at ₹ 5.70 per metre.

9. The area of a square park is the same as that of a rectangular park. If the side of the square park is 60 m and the length of the rectangular park is 90 m, find the breadth of the rectangular park.

10. A wire is in the shape of a rectangle. Its length is 40 cm and breadth is 22 cm. If the same wire is rebent in the shape of a square, what will be the measure of each side? Also find which shape encloses more area and by how much?

11. A door of breadth 1m and height 2 m is fitted in a wall. The length of the wall is 4.5 m and the height is 3.6 m. Find the cost of white washing the wall, if the rate of white washing the wall is ₹ 20 per m^2.

12. A rectangular park is 45 m long and 30 m wide. A path 2.5 m wide is constructed outside the park. Find the area of the path.

13. A garden is 90 m long and 75 m broad. A path 5 m wide is to be built outside and around it. Find the area of the path. Also find the area of the garden in hectare.

14. A picture is painted on a cardboard 8 cm long and 5 cm wide such that there is a margin of 1.5 cm along each of its side. Find the total area of the margin.

15. A *verandah* of width 2.25 m is constructed all along outside a room which is 5.5 m long and 4 m wide. Find:

(*i*) the area of the *verandah*.

(*ii*) the cost of cementing the floor of the *verandah* at the rate of ₹ 200 per m^2.

16. A path 5 m wide runs along inside a square park of side 100 m. Find the area of the path. Also find the cost of cementing it at the rate of ₹ 250 per 10 m^2.