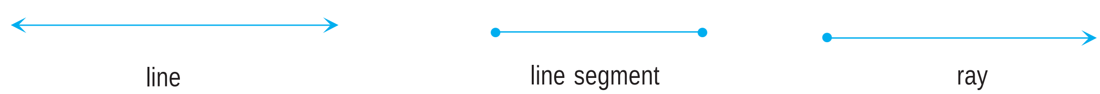
9 Lines and Angles

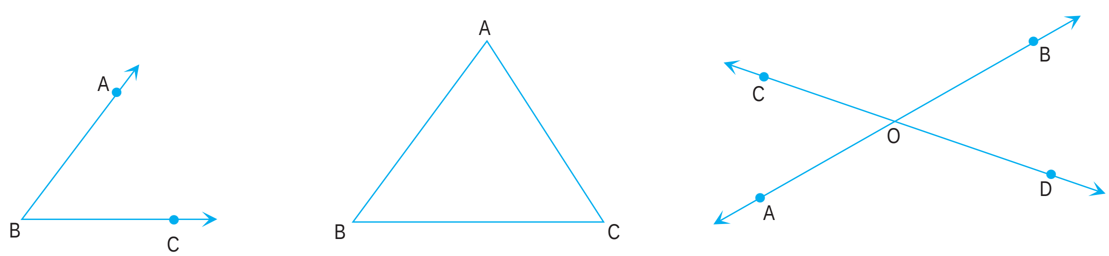
INTRODUCTION

In class VI, you have learnt some basic concepts and terms of geometry – point, line, plane, line segment, ray, angle and types of angles. In this chapter, we shall learn about some pairs of related angles – complementary angles, supplementary angles, adjacent angles, linear pair, sum of angles at a point, sum of angles at a point on one side of a straight line and vertically opposite angles. We shall also learn about pairs of intersecting and parallel lines, transversal, angles made by a transversal and properties of angles associated with a pair of parallel lines.

Recall, the basic concept of a line is its straightness and it extends indefinitely in both directions *i.e.* it has no definite length and it has no end points. A line segment is a portion of a line. It has two end points and a definite length. A ray is a part of a line that extends only in one direction. It has one end point (called initial point) and has no definite length.



We know that when two rays or two line segments or two lines meet, an angle is formed. The common point is called the vertex of the angle. Look at the figures given below:



(*i*) (*ii*) (*iii*)

In fig. (*i*), two ray $\overrightarrow{AB}$ and $\overrightarrow{BC}$ meet at B, they form angle ABC. Point B is the vertex and two rays $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are the arms (or sides) of the angle. The angle ABC is denoted by $\angle ABC$.

In fig. (*ii*), the pairs of line segments AB, BC; BC, CA; CA, AB meet at the points B, C and A respectively. Three angles formed by these pairs of line segments are $\angle ABC$, $\angle BCA$ and $\angle CAB$ respectively.

In fig. (*iii*), the pair of lines AB and CD meet at the point O. Four angles formed are $\angle AOD$, $\angle DOB$, $\angle BOC$ and $\angle COA$.

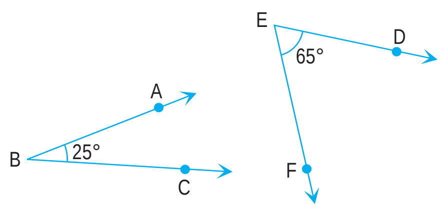
The *size* (magnitude or measure) of an angle is the amount by which one of the arms needs to be rotated about the vertex so that it lies on the top of the other arm. To measure an angle, one complete turn is divided into 360 equal parts and each part is called one degree and is written as $1°$. We shall measure angles in degrees. Usually, the measure of $\angle ABC$ *i.e.* $m\angle ABC$ is written as $\angle ABC$. Two angles are called *equal* if they have same measure.

RELATED ANGLES

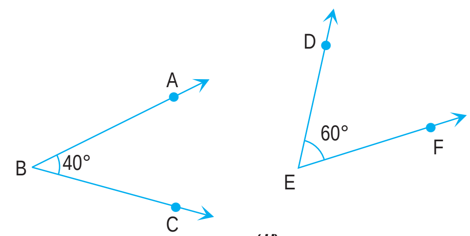
Complementary angles

Two angles are called ***complementary angles*** *if the sum of their measures is 90°. Each angle is called the complement of the other.*

Look at the figures given below:



(*i*)



(*ii*)

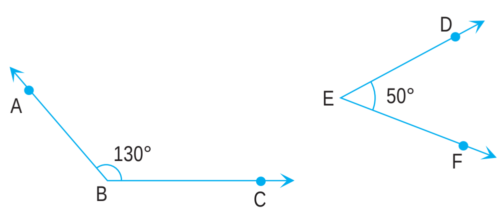
In fig. (*i*), ∠ABC + ∠DEF = 25° + 65° = 90° *i.e.* sum of their measures is 90°, therefore, ∠ABC and ∠DEF are complementary.

In fig. (*ii*), ∠ABC + ∠DEF = 40° + 60° = 100° *i.e.* sum of their measures is not 90°, therefore, ∠ABC and ∠DEF are not complementary.

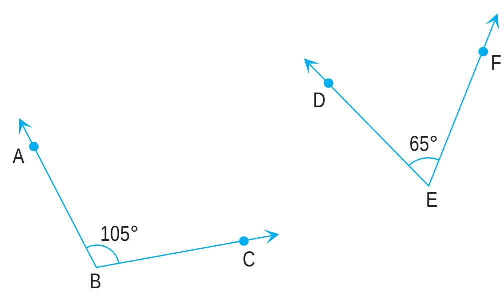
Supplementary angles

Two angles are called ***supplementary angles*** *if the sum of their measures is 180°. Each angle is called the supplement of the other.*

Look at the figures given below:



(*i*)



(*ii*)

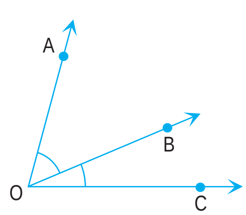
In fig. (*i*), ∠ABC + ∠DEF = 130° + 50° = 180° *i.e.* sum of their measures is 180°, therefore, ∠ABC and ∠DEF are supplementary.

In fig. (*ii*), ∠ABC + ∠DEF = 105° + 65° = 170° *i.e.* sum of their measures is not 180°, therefore, ∠ABC and ∠DEF are not supplementary.

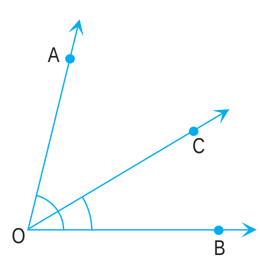
Adjacent angles

Two angles are called ***adjacent angles*** *if they have a common vertex, a common arm and their non-common arms lie on either side of the common arm.*

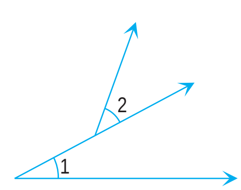
Look at the figures given below:



(*i*)



(*ii*)



(*iii*)

In fig. (*i*), ∠AOB and ∠BOC have a common vertex O, common arm OB and the non-common arms OA and OC lie on either (opposite) sides of the common arm OB, therefore, ∠AOB and ∠BOC are adjacent angles.

In fig. (*ii*), ∠AOB and ∠BOC have a common vertex O, common arm OB but the non-common arms OA and OC lie on the same side of the common arm OB, therefore, ∠AOB and ∠BOC are not adjacent angles.

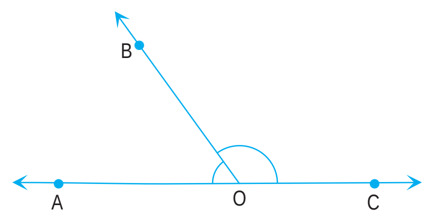
In fig. (*iii*), ∠1 and ∠2 do not have a common vertex, therefore, ∠1 and ∠2 are not adjacent angles.

Note. Two adjacent angles have no common interior points.

Linear pair

*Two adjacent angles are said to form a **linear pair** if their non-common arm are opposite rays i.e. they are in a straight line.*

In the adjoining figure, ∠AOB and ∠BOC are two adjacent angles and their non-common arms OA and OC are the opposite rays, therefore, ∠AOB and ∠BOC form a linear pair. As OA and OC are opposite rays, OA and OC are in a straight line.



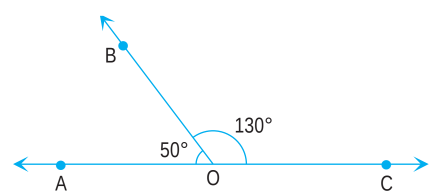
∴ ∠AOC = 180°.

From given fig., ∠AOC = ∠AOB + ∠BOC

⇒ ∠AOB + ∠BOC = 180°.

Thus, two angles in a linear pair are supplementary. Conversely, if two adjacent angles are supplementary *i.e.* if the sum of their measures is 180°, then the non-common arms are in a straight line and hence they form a linear pair.

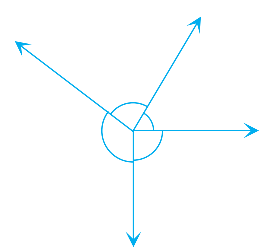
In the adjoining figure, ∠AOB and ∠BOC are two adjacent angles such that ∠AOB + ∠BOC = 50° + 130° = 180°, therefore, ∠AOB and ∠BOC form a linear pair.



Note. If a pair of supplementary angles are placed adjacent to each other, then they form a linear pair.

Angles at a point

In the adjoining diagram, the four angles together make one complete turn, so they add upto 360°.



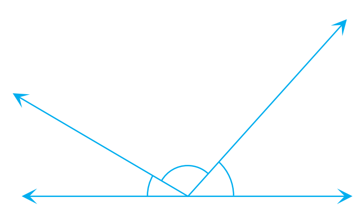
This is true no matter how many angles are formed at a point.

Thus:

Sum of angles at a point = 360°.

Angles on one side of a straight line

In the adjoining diagram, the three angles together make a straight line, so they add upto 180°.



This is true no matter how many angles make up the straight line. Thus:

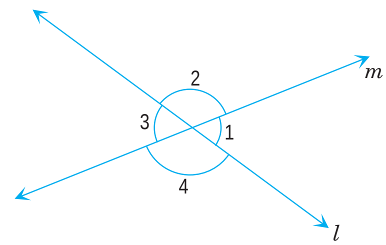
Sum of angles at a point on one side of a straight line = 180°.

Vertically opposite angles

When two straight lines intersect each other, they form four angles at their point of intersection say ∠1, ∠2, ∠3 and ∠4.

∠1 and ∠3 are called **vertically opposite angles** to each other and so are ∠2 and ∠4.

They are called vertically opposite angles because they have the same vertex and are opposite to each other. In fact, vertically opposite angles are formed by the non-common arms.



ACTIVITY 6

Vertically opposite angles are equal

Materials required

(*i*) A sheet of white paper
(*ii*) Ruler
(*iii*) Tracing paper
(*iv*) Pin

Steps

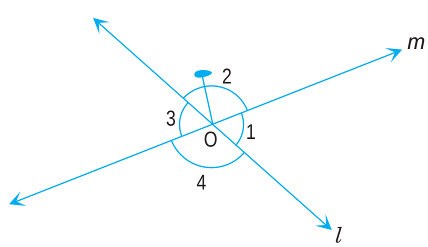
1. On a sheet of paper, draw two straight lines *l* and *m* intersecting at the point O. Four angles are formed at the point O, say ∠1, ∠2, ∠3 and ∠4.

∠1, ∠3 form one pair of vertically opposite angles and ∠2, ∠4 form another pair of vertically opposite angles.

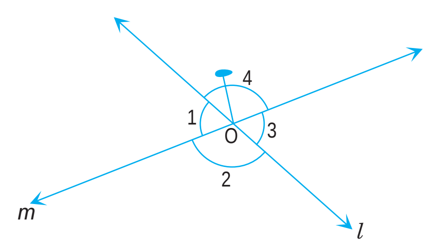
2. Make a trace copy (replica) of the fig. (*i*) on a tracing paper.

3. Place the trace copy on the original figure such that one of the angles match its copy, then the other angles will match the copy.

4. Fix a pin at the point O and rotate the copy through 180°.



(*i*)



(*ii*)

Observation

The lines of the copy will coincide with the original figure (as shown in fig. (*ii*)).

We note that ∠1 coincides with ∠3 and ∠2 coincides with ∠4.

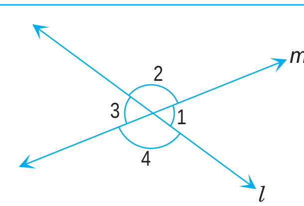
Result

∠1 = ∠3 and ∠2 = ∠4.

If follows that vertically opposite angles are equal.

Vertically opposite angles are equal

When two straight line intersect each other, they form four angles at their point of intersection, say ∠1, ∠2, ∠3 and ∠4.



Look at the figure, ∠1 and ∠2 form a linear pair.

$\therefore \quad \angle 1 + \angle 2 = 180°$ ....(*i*)

Again from figure, $\angle 3$ and $\angle 2$ form a linear pair.

$\therefore \quad \angle 3 + \angle 2 = 180°$ ....(*ii*)

Form (*i*) and (*ii*), we get

$\angle 1 + \angle 2 = \angle 3 + \angle 2 \Rightarrow \angle 1 = \angle 3.$

In the same way, we can show that $\angle 2 = \angle 4$.

Hence, vertically opposite angles are equal.

Example 1. (*i*) Can two acute angles be complement to each other?

(*ii*) Can two obtuse angles be complement to each other?

(*iii*) Can two acute angles be supplementary?

(*iv*) Can two adjacent obtuse angles form a linear pair?

Solution.

(*i*) Yes; pairs of angles like 30° and 60°; 25° and 65° are complements of each others.

(*ii*) No; as the sum of two obtuse angles is always greater than 180°, so they can never be complement of each other.

(*iii*) No; as the sum of two acute angles is always less than 180°, so they can never be supplementary angles.

(*iv*) No; as the sum of two obtuse angles is always greater than 180°, so they cannot form a linear pair.

Example 2. In the given figure, straight lines AB and CD intersect each other at O:

(*i*) Is $\angle 1$ adjacent to $\angle 2$?

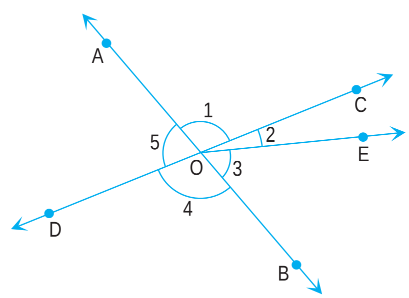
(*ii*) Is $\angle AOC$ adjacent to $\angle AOE$?

(*iii*) Do $\angle COE$ and $\angle EOD$ form a linear pair?

(*iv*) Are $\angle BOD$ and $\angle DOA$ supplementary?

(*v*) Is $\angle 1$ vertically opposite to $\angle 4$?

(*vi*) What is the vertically opposite angle of $\angle 5$?



Solution.

(*i*) Yes; it is clear from figure.

(*ii*) No; OA is the common arm of $\angle AOC$ and $\angle AOE$ but their non-common arms OC and OE lie on the same side of the common arm OA, therefore, $\angle AOC$ and $\angle AOE$ are not adjacent angles.

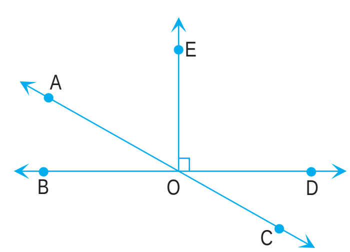
(*iii*) Yes; because $\angle COE$ and $\angle EOD$ are adjacent angles and their non-common arms are in a straight line.

(*iv*) Yes; as AB is a straight line, $\angle BOD + \angle DOA = 180°$, therefore these angles are supplementary.

(*v*) Yes, it is clear from figure.

(*vi*) $\angle BOC$ is vertically opposite to $\angle 5$.

Example 3. In the adjoining figure, straight lines AC and BD intersect each other at O and $\overrightarrow{OE} \perp \overleftrightarrow{BD}$. Name the following pairs of angles:



(*i*) obtuse vertically opposite angles

(*ii*) adjacent complementary angles

(*iii*) equal supplementary angles

(*iv*) unequal supplementary angles

(*v*) adjacent angles that do not form a linear pair.

Solution.

(*i*) ∠BOC and ∠AOD

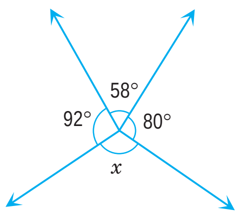
(*ii*) ∠EOA and ∠AOB

(*iii*) ∠BOE and ∠EOD

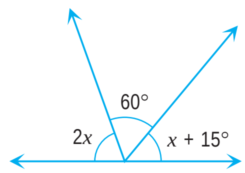
(*iv*) Pairs of unequal supplementary angles are: ∠BOA, ∠AOD; ∠AOD, ∠DOC; ∠DOC, ∠COB; ∠COB, ∠BOA, ∠AOE, ∠EOC

(*v*) Pair of adjacent angles that do not form a linear pair are: ∠AOB, ∠AOE; ∠AOE, ∠EOD; ∠EOD, ∠DOC.

Example 4. Find the value of x in each of the following diagrams:

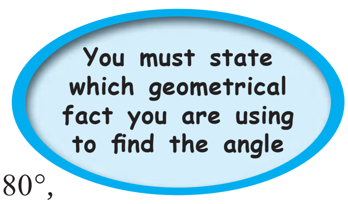


(*i*)



(*ii*)

Solution.



(*i*) As the sum of angles at a point = 360°,

$x + 80° + 58° + 92° = 360°$

$\Rightarrow \quad x = 360° - 80° - 58° - 92° = 130°$

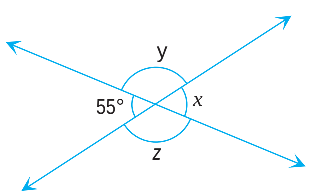
(*ii*) As the sum of angles at a point on one side of a straight line is 180°,

$x + 15° + 60° + 2x = 180°$

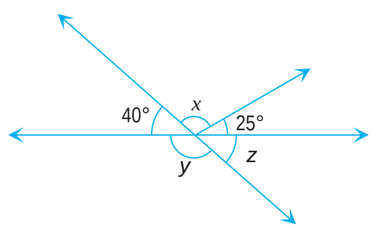
$\Rightarrow \quad 3x = 180° - 15° - 60° = 105°$

$\Rightarrow \quad x = 35°.$

Example 5. Find the values of x, y and z in each of the following diagrams:



(*i*)



(*ii*)

Solution.

(*i*) $x = 55°$ (vertically opposite angles)

$55° + y = 180°$ (linear pair)

$\Rightarrow \quad y = 180° - 55° \Rightarrow y = 125°$

$z = y$ (vertically opposite angles)

$\Rightarrow \quad z = 125°.$

(*ii*) As the sum of angles at a point on one side of a straight line is 180°,

$$40° + x + 25° = 180° \Rightarrow x = 180° - 40° - 25°$$

$$\Rightarrow \quad x = 115°$$

$$40° + y = 180° \quad \text{(linear pair)}$$

$$\Rightarrow \quad y = 180° - 40° \Rightarrow y = 140°$$

$$z = 40° \quad \text{(vertically opposite angles)}$$

Example 6. If the difference in the measures of two complementary angles is 12°, then find the measures of the angles.

Solution. Let one angle be $x°$, then the other angle is $(x + 12)°$.

As the given angles are complementary angles,

$$x + (x + 12) = 90$$

$$\Rightarrow \quad 2x = 90 - 12 \Rightarrow 2x = 78$$

$$\Rightarrow \quad x = 39 \text{ and } x + 12 = 39 + 12 = 51.$$

Hence, the measures of the required angles are 39° and 51°.

Example 7. Two complementary angles are in the ratio 2 : 3, find these angles.

Solution. Since the given angles are in the ratio 2 : 3, let the angles be $2x$ and $3x$.

As the given angles are complementary angles,

$$2x + 3x = 90° \Rightarrow 5x = 90°$$

$$\Rightarrow \quad x = 18°$$

$$\Rightarrow \quad 2x = 36° \text{ and } 3x = 54°.$$

Hence, the required angles are 36° and 54°.

Example 8. If two angles are supplementary angles and one angle is 30° less than twice the other, find the angles.

Solution. Let one angle be $x°$, then the other angle $= (2x - 30)°$.

As the given angles are supplementary angles,

$$x + (2x - 30) = 180$$

$$\Rightarrow \quad 3x = 180 + 30 = 210$$

$$\Rightarrow \quad x = 70 \text{ and } 2x - 30 = 2 \times 70 - 30 = 110.$$

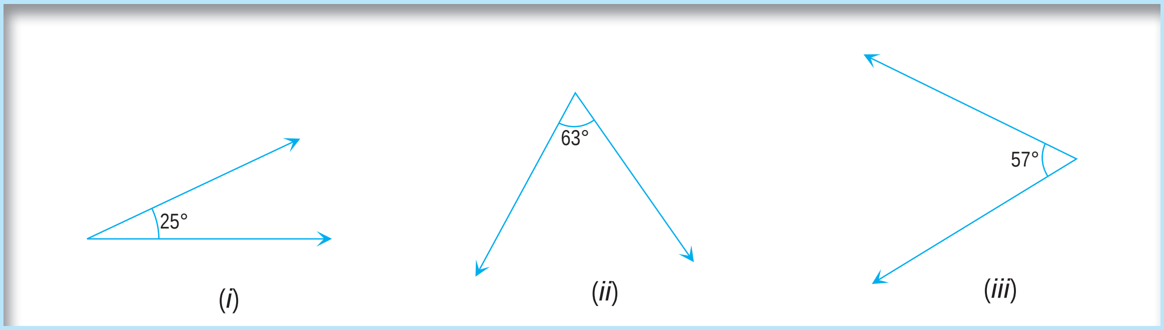
Hence, the required angles are 70° and 110°.

Exercise 9.1

1. (*i*) Can two right angles be complementary?
   (*ii*) Can two right angles be supplementary?
   (*iii*) Can two adjacent angles be complementary?
   (*iv*) Can two adjacent angles be supplementary?
   (*v*) Can two obtuse angles be adjacent?
   (*vi*) Can an acute angle be adjacent to an obtuse angle?
   (*vii*) Can two right angles form a linear pair?

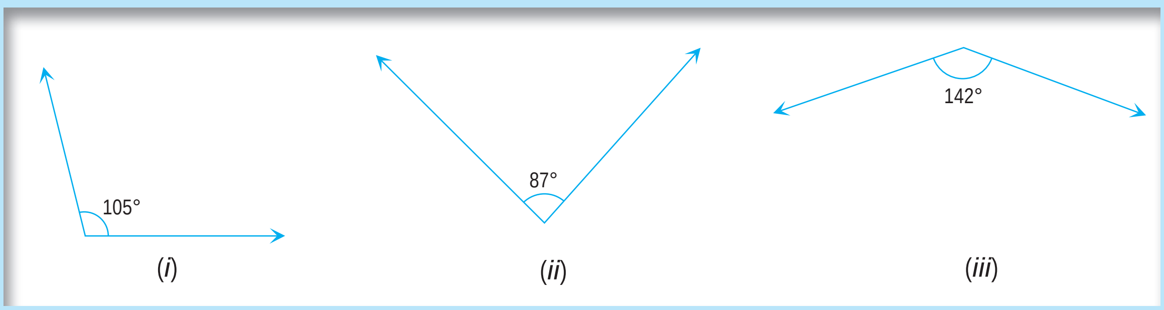
2. Find the measure of the complement of each of the following angles:
(*i*) 45° (*ii*) 65° (*iii*) 41° (*iv*) 54°

3. Find the complement of each of the following angles:



4. Find the measure of the supplement of each of the following angles:
(*i*) 100° (*ii*) 90° (*iii*) 55° (*iv*) 135°

5. Find the supplement of each of the following angles:



6. Identify which of the following pairs of angles are complementary and which are supplementary:
(*i*) 65°, 115° (*ii*) 63°, 27° (*iii*) 130°, 50°
(*iv*) 112°, 68° (*v*) 45°, 45° (*vi*) 72°, 18°.

7. (*i*) Find the angle which is equal to its complement.
(*ii*) Find the angle which is equal to its supplement.

8. Two complementary angles are $(x + 4)°$ and $(2x – 7)°$, find the value of x.

9. Two supplementary angles are $(x + 38)°$ and $(3x – 58)°$, find x.

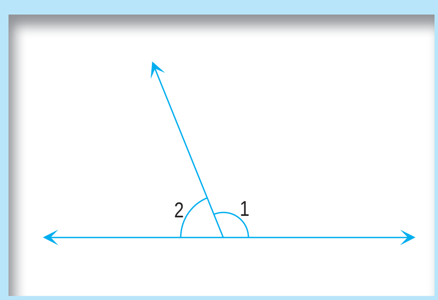
10. Two supplementary angles are in the ratio of 2 : 7, find the angles.

11. Among two supplementary angles, the measure of the longer angle is 44° more than the measure of the smaller angle. Find their measures.

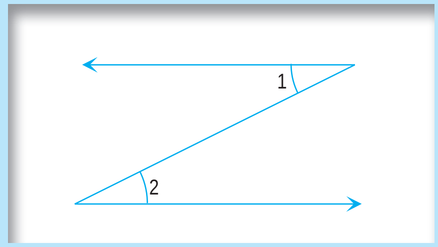
12. If an angle is half of its complement, find the measure of angles.

13. An angle is greater than 45°. Is its complementary angle greater than 45° or equal to 45° or less than 45°?

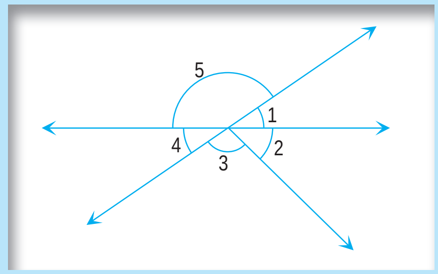
14. In the adjoining figure, $\angle 1$ and $\angle 2$ are supplementary. If $\angle 1$ is decreased, what change should take place in $\angle 2$ so that both angles still remain supplementary?



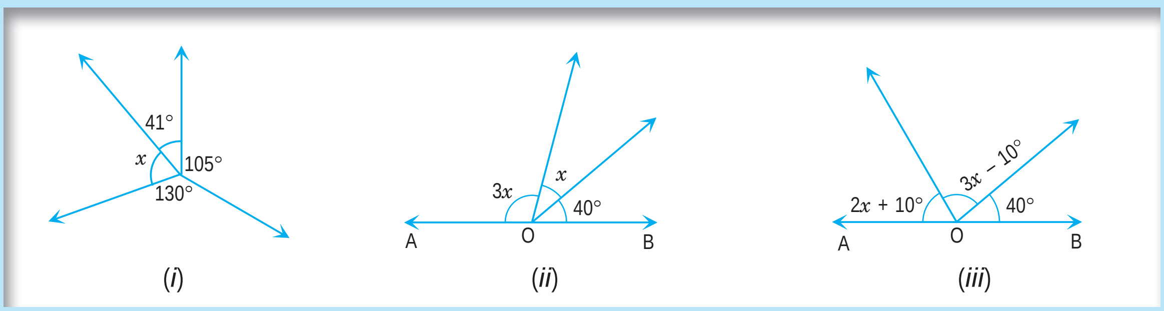
15. In the adjoining figure, is $\angle 1$ adjacent to $\angle 2$? Give reasons.



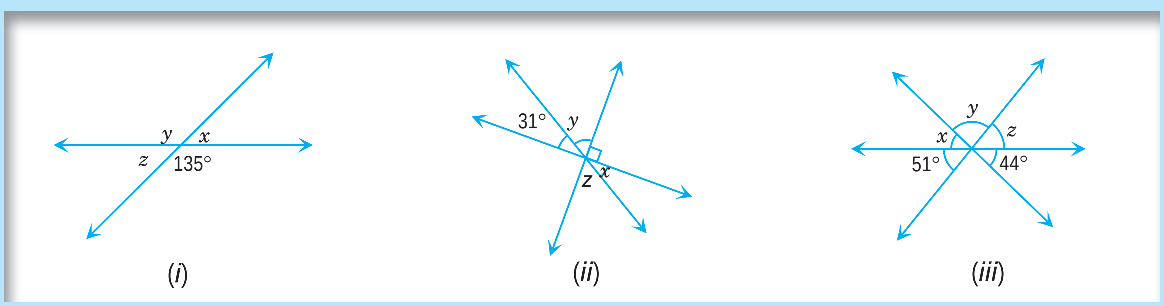
16. In the adjoining figure, write pairs of angles which are:
(*i*) vertically opposite angles
(*ii*) linear pairs



17. Find the value of x in each of the following diagrams:



18. Find the values of x, y and z in each of the following diagrams:



PAIRS OF LINES

Intersecting lines

Two lines l and m are ***intersecting lines*** *if they have a point in common.*

In the adjoining figure, lines l and m intersect each other at the point O. The point O is called the **point of intersection.**