6 Linear Equations

INTRODUCTION

In class VI, we have learnt about simple algebraic equation in one variable and found their solution by trial and error method. In this chapter, we shall learn direct and practical method of finding a solution of a linear equation in one variable. We shall also learn the uses of linear equation in solving some real life (word) problems.

LINEAR EQUATIONS

An (algebraic) equation is a statement that two expressions are equal. It may involve one or more than one unknowns (variables, literal numbers).

Thus, $2x - 3 = 1 + x$, $p^2 + p + 1 = 3$ are equations in one variable, $2x + 3y = 13$, $x^2 + y^2 + 2xy = 4$ are equations in two variables.

An equation has two sides separated by the sign '='. The side on the left of equal sign is called left hand side (L.H.S.) and the side on the right of equal sign is called right hand side (R.H.S.). An equation remains same if its L.H.S. and R.H.S. are interchanged. If there is some sign other than equality sign (=) between L.H.S. and R.H.S., then it is not an equation.

Thus, $2x + 5 > 9$ and $3x - 2 < 7$ are not equations.

An equation containing only one variable (literal) with highest power 1 is called a ***linear equation*** *in that variable.*

For example:

$$3x + 5 = 8, \quad 3 - 2x = 5x + 1 \text{ and } 4n = \frac{2}{3}n - 1$$

are all linear equations in one variable.

In this chapter, we shall take up linear equations in one variable only.

Solving a linear equation in one variable

A number which satisfies the given linear equation is called a ***solution*** *or* ***root*** *of the equation.*

'Satisfying the equation' means that if the variable (literal) involved in the equation is replaced by the number, then both sides of the equation become equal.

The process of finding the particular value of the variable (literal) which makes both sides of the equation equal is called **solving the equation.**

Consider the equation $x + 3 = 5$.

What particular value of x would make this equation true?

We can try to solve it by **trial and error** method.

Let us try $x = 0$. Then left hand side (L.H.S.) $= 0 + 3 = 3$, while right hand side (R.H.S.) $= 5$. So $x = 0$ is not a solution.

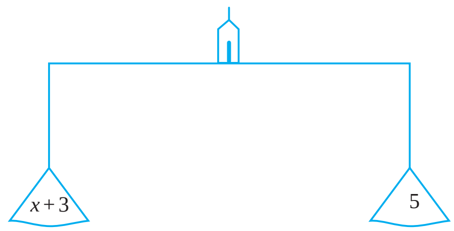
Now let us try $x = 1$. Then L.H.S. = 1 + 3 = 4 while R.H.S. = 5. So $x = 1$ is not a solution.

Now let us try $x = 2$. Then L.H.S. = 2 + 3 = 5 and R.H.S. = 5.

So $x = 2$ is a solution of the equation $x + 3 = 5$.

You can immediately see that this method of trial and error can be time consuming. Let us find a much better and practical method.

You must be familiar with how a weighing balance works. When the weights on two sides are equal, the balance is balanced (what else?). Then you can add equal weights to both sides and still the two sides will hang in balance. You can remove equal weights from both sides, and still the two sides will hang in balance. We can apply these ideas to algebraic equations.



Rules for solving linear equations

Rule 1. If equals are added to equals, the sums are equals. In other words, you can add the same number to both sides of an equation.

For example, if $x + 3 = 5$, then $x + 3 + 2 = 5 + 2$.

Rule 2. If equals are subtracted from equals, the remainders are equal. In other words, you can subtract the same number from both sides of an equation.

For example, if $2x + 1 = 11$, then $2x + 1 - 1 = 11 - 1$.

Rule 3. The two sides of an equation may be multiplied by same non-zero number.

For example, if $\frac{x}{5} = 7$, then $\frac{x}{5} \times 5 = 7 \times 5$.

Rule 4. The two sides of an equation may be divided by same non-zero number.

For example, if $3x = 12$, then $\frac{3x}{3} = \frac{12}{3}$.

Now, let us solve a number of linear equations using these rules.

Example 1. Solve the following equations:

(*i*) $x + 3 = 7$ (*ii*) $x - 2 = 9$ (*iii*) $\frac{x}{3} = 7$ (*iv*) $4x = 20$.

Solution.

(*i*) Given $x + 3 = 7$.

Subtracting 3 from both sides, we get

$$x + 3 - 3 = 7 - 3 \Rightarrow x = 4.$$

(*ii*) Given $x - 2 = 9$.

Adding 2 to both sides, we get

$$x - 2 + 2 = 9 + 2 \Rightarrow x = 11.$$

(*iii*) Given $\frac{x}{3} = 7$.

Multiplying both sides by 3, we get

$$\frac{x}{3} \times 3 = 7 \times 3 \Rightarrow x = 21.$$

(*iv*) Given $4x = 20.$

Dividing both sides by 4, we get

$$\frac{4x}{4} = \frac{20}{4} \Rightarrow x = 5.$$

Example 2. Solve $2x - \frac{1}{2} = \frac{11}{2}$ and verify your answer.

Solution. Given $2x - \frac{1}{2} = \frac{11}{2}$

$\Rightarrow$ $2x - \frac{1}{2} + \frac{1}{2} = \frac{11}{2} + \frac{1}{2}$ (adding $\frac{1}{2}$ to both sides)

$\Rightarrow$ $2x = \frac{11+1}{2} = \frac{12}{2}$

$\Rightarrow$ $2x = 6$

$\Rightarrow$ $\frac{2x}{2} = \frac{6}{2}$ (dividing both sides by 2)

$\Rightarrow$ $x = 3.$

Verification:

Putting $x = 3$, L.H.S. $= 2 \times 3 - \frac{1}{2} = 6 - \frac{1}{2} = \frac{12-1}{2} = \frac{11}{2}$, which is equal to R.H.S.

Example 3. Solve the following equations:

(*i*) $3(y + 7) = 15$ (*ii*) $\frac{5z+1}{3} = 7.$

Solution.

(*i*) Given $3(y + 7) = 15$

$\Rightarrow$ $\frac{3(y+7)}{3} = \frac{15}{3}$ (dividing both sides by 3)

$\Rightarrow$ $y + 7 = 5$

$\Rightarrow$ $y + 7 - 7 = 5 - 7$ (subtracting 7 from both sides)

$\Rightarrow$ $y = -2.$

(*ii*) Given $\frac{5z+1}{3} = 7$

$\Rightarrow$ $\frac{5z+1}{3} \times 3 = 7 \times 3$ (multiplying both sides by 3)

$\Rightarrow$ $5z + 1 = 21$

$\Rightarrow$ $5z + 1 - 1 = 21 - 1$ (subtracting 1 from both sides)

$\Rightarrow$ $5z = 20$

$\Rightarrow$ $\frac{5z}{5} = \frac{20}{5}$ (dividing both sides by 5)

$\Rightarrow$ $z = 4.$

Example 4. Solve $2n + 16 = 4 - n.$

Solution. Given $2n + 16 = 4 - n$

$\Rightarrow$ $2n + 16 + n = 4 - n + n$ (adding n to both sides)

$\Rightarrow$ $3n + 16 = 4$

$\Rightarrow$ $3n + 16 - 16 = 4 - 16$ (subtracting 16 from both sides)

$\Rightarrow \quad 3n = -12$

$\Rightarrow \quad \frac{3n}{3} = \frac{-12}{3}$ (dividing both sides by 3)

$\Rightarrow \quad n = -4.$

Rule of transposition

A term may be transposed from one side of the equation to the other side, but its sign will change.

For example:

(*i*) $3x + 5 = 8 \Rightarrow 3x = 8 - 5$

(transposing + 5 from L.H.S. to R.H.S. and changing its sign)

(*ii*) $2n = 7 - n \Rightarrow 2n + n = 7$

(transposing $-n$ from R.H.S. to L.H.S. and changing its sign)

Example 5. Solve the following equations:

(*i*) $4x - 7 = 13$ (*ii*) $\frac{x}{5} + 3 = 1$ (*iii*) $\frac{5x}{3} + 3 = x + 7.$

Solution.

(*i*) Given $4x - 7 = 13$

$\Rightarrow \quad 4x = 13 + 7$ (transposing –7 from L.H.S. to R.H.S.)

$\Rightarrow \quad 4x = 20$

$\Rightarrow \quad \frac{4x}{4} = \frac{20}{4}$ (dividing both sides by 4)

$\Rightarrow \quad x = 5.$

(*ii*) Given $\frac{x}{5} + 3 = 1$

$\Rightarrow \quad \frac{x}{5} = 1 - 3$ (transposing 3 from L.H.S. to R.H.S.)

$\Rightarrow \quad \frac{x}{5} = -2$

$\Rightarrow \quad \frac{x}{5} \times 5 = (-2) \times 5$ (multiplying both sides by 5)

$\Rightarrow \quad x = -10.$

(*iii*) Given $\frac{5x}{3} + 3 = x + 7$

$\Rightarrow \quad \frac{5x}{3} = x + 7 - 3$ (transposing 3 from L.H.S. to R.H.S.)

$\Rightarrow \quad \frac{5x}{3} = x + 4$

$\Rightarrow \quad \frac{5x}{3} - x = 4$ (transposing x from R.H.S. to L.H.S.)

$\Rightarrow \quad \left(\frac{5}{3} - 1\right)x = 4$

$\Rightarrow \quad \frac{5-3}{3}x = 4 \Rightarrow \frac{2}{3}x = 4$

$\Rightarrow \quad \frac{3}{2} \times \frac{2}{3}x = \frac{3}{2} \times 4$ (multiplying both sides by $\frac{3}{2}$)

$\Rightarrow \quad x = 6.$

Example 6. Solve $3(x-2) = 2(x+1) - 3$.

Solution. Here we should remove the brackets first.

$$3(x-2) = 2(x+1) - 3$$

$\Rightarrow \quad 3x - 6 = 2x + 2 - 3$ (removing brackets)

$\Rightarrow \quad 3x = 2x + 2 - 3 + 6$ (transposing -6 from L.H.S. to R.H.S.)

$\Rightarrow \quad 3x = 2x + 5$

$\Rightarrow \quad 3x - 2x = 5$ (transposing $2x$ from R.H.S. to L.H.S.)

$\Rightarrow \quad x = 5.$

Note. From the above examples, we observe that every linear equation in one variable has exactly one solution.

Example 7. Find the value of m if the value of the expression $2x^3 - 5x^2 + mx - 7$ is equal to 10 when $x = -1$.

Solution. Given, the value of $2x^3 - 5x^2 + mx - 7$ is 10 when $x = -1$.

$\therefore \quad 2(-1)^3 - 5(-1)^2 + m(-1) - 7 = 10$

$\Rightarrow \quad -2 - 5 - m - 7 = 10 \Rightarrow -14 - m = 10$

$\Rightarrow \quad -m = 10 + 14 \Rightarrow -m = 24 \Rightarrow m = -24.$

Exercise 6.1

1. Solve each of the following equations:

(*i*) $x + 6 = 8$ (*ii*) $x + 6 = 4$ (*iii*) $2 + x = 5$

(*iv*) $2 - x = 5$ (*v*) $4x = 8$ (*vi*) $4x = -6$

(*vii*) $\frac{x}{2} = 5$ (*viii*) $2y - 3 = 2$ (*ix*) $4 - 5y = 2.$

2. Solve the following equations:

(*i*) $5(x+1) = 25$ (*ii*) $2(3x-1) = 10$ (*iii*) $\frac{3x-1}{4} = 11.$

3. Solve the following equations:

(*i*) $2x - 3 = 1 + x$ (*ii*) $5x - 6 = 12 - x$ (*iii*) $6y - 1 = 2y + 1$

(*iv*) $\frac{n}{3} + 1 = 4 - n$ (*v*) $5p + 7 = 19 - 2p$ (*vi*) $2x + \frac{5}{2} = \frac{2}{3} - x$

(*vii*) $\frac{x}{2} - 5 = \frac{x}{3} - 4$ (*viii*) $18 - \frac{3y}{4} = 11 + y.$

4. Solve the following equations and verify your answers:

(*i*) $3(x+7) = 18$ (*ii*) $2(x-1) = x + 2$

(*iii*) $3x - \frac{1}{3} = 2\left(x - \frac{1}{2}\right) + 5$ (*iv*) $4(2x-1) - 2(x-5) = 5(x+1) + 3.$

5. Find the value of a if the value of $2x^2 - x - a$ is equal to 5 when $x = 0$.

6. Find the value of p if the value of $x^4 - 3x^3 - px - 5$ is equal to 23 when $x = -2$.

USES OF LINEAR EQUATIONS

Word problems

Problems stated in words are called ***word*** *or* ***applied problems****.*

Success with word (or applied) problems comes with practice. Solving word problem involves two steps. First, translating the words of the problem into an algebraic equation. Second, solving the resulting equation.

Solving word problems

Due to the wide variety of word (or applied) problems, there is no single technique that works in all problems. However, the following general suggestions may prove helpful.

- Read the statement of the problem carefully and determine what quantity must be found.
- Represent the unknown quantity by a letter.
- Determine which expressions are equal and write an equation.
- Solve the resulting equation.

Example 1. If 5 is added to twice a number, the result is 29. Find the number.

Solution. Let the required number be x.

Twice the number = $2x$,

5 added to twice the number = $2x + 5$.

According to the problem,

$$2x + 5 = 29$$

$$\Rightarrow \quad 2x = 29 - 5 \Rightarrow 2x = 24$$

$$\Rightarrow \quad x = 12.$$

Hence, the required number is 12.

Example 2. If thrice a certain number is diminished by 7, the result is 9 more than the number. Find the number.

Solution. Let the required number be x.

Thrice the number = $3x$.

Thrice the number diminished by 7 = $3x - 7$,

9 more than the number = $x + 9$.

According to the problem,

$$3x - 7 = x + 9$$

$$\Rightarrow \quad 3x - x = 9 + 7 \Rightarrow 2x = 16 \Rightarrow x = 8.$$

Hence, the required number is 8.

Example 3. One-fourth of a number exceeds one-fifth of its succeeding number by 3; find the number.

Solution. Let the required number be x.

Its succeeding number = $x + 1$.

According to the problem,

$$\frac{1}{4}x - \frac{1}{5}(x + 1) = 3$$

$$\Rightarrow \quad 5x - 4(x + 1) = 60 \quad \text{(multiplying by 20)}$$

$$\Rightarrow \quad 5x - 4x - 4 = 60$$

$$\Rightarrow \quad x = 60 + 4 \Rightarrow x = 64.$$

Hence, the required number is 64.