4 Exponents and Powers

INTRODUCTION

We know that the radius of the earth is approximately 6366000 m, the distance between the sun and the earth is approximately 149,600,000,000 m and the speed of light in vacuum is approximately 299,800,000 m/s.

Such large numbers are difficult to read, understand and compare. To make these numbers easy to read, understand and compare, we use exponents. In this chapter, we shall learn about exponents and their laws and also learn how to use them.

EXPONENTS

Repeated multiplication by the same number is very common.

For example:

(*i*) $2 \times 2 \times 2 \times 2 \times 2$. It is usually written as 2^5.

Thus, $2 \times 2 \times 2 \times 2 \times 2 = 2^5$, where the repeated factor 2 is called the **base** and 5 is called the **exponent** or **index**. 2^5 is read as '2 raised to the power 5' or '2 to the power 5' or simply '2 power 5'. Note that $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$ *i.e.* $32 = 2^5$. 2^5 is called the **exponential form** of 32.

We have:

$$32 = \boxed{2}^{\boxed{5}} \quad \begin{array}{l} \leftarrow \text{exponent or index} \\ \leftarrow \text{base} \end{array}$$

(*ii*) $3 \times 3 \times 3 \times 3 = 3^4$, where 3 is the base and 4 is the index.

(*iii*) $(-4) \times (-4) \times (-4) = (-4)^3$, where base $= -4$ and index $= 3$.

(*iv*) $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \left(\frac{2}{3}\right)^4$, where base $= \frac{2}{3}$ and index $= 4$.

This leads to:

If a is any rational number and n is a natural number, then

$$a^n = a \times a \times a \text{.... multiplied } n \text{ times,}$$

*where a is called the **base** and n is called the **exponent** or **index** and a^n is the **exponential form**.*

a^n is read as 'a raised to the power n' or 'a to the power n' or simply 'a power n.'

In particular, $a^1 = a$.

For example:

$10^4 = 10 \times 10 \times 10 \times 10$ *i.e.* $10^4 = 10000$;

here base $= 10$, exponent (or index) $= 4$ and 10^4 is the exponential form of the number 10000.

Some powers have special names

If the power of a number is 2 then it is named as square and if the power of a number is 3 then it is named as cube. Thus,

$9 = 3 \times 3 = 3^2$ [read as 3 squared or 3 raised to the power 2]

$100 = 10 \times 10 = 10^2$ [read as 10 squared or 10 raised to power 2]

$125 = 5 \times 5 \times 5 = 5^3$ [read as 5 cubed or 5 raised to power 3]

$1000 = 10 \times 10 \times 10 = 10^3$ [read as 10 cubed or 10 raised to power 3]

Note: $a \times b$ is written as ab, read as a into b.

$a \times a \times a \times b \times b$ is written as a^3b^2, read as a cubed into b squared etc.

Example 1. Write the base and exponent for the following exponential form:

(*i*) 5^3 (*ii*) 7^5 (*iii*) $(-3)^6$.

Solution.

(*i*) For 5^3, base = 5 and exponent = 3.

(*ii*) For 7^5, base = 7 and exponent = 5.

(*iii*) For $(-3)^6$, base = -3 and exponent = 6.

Example 2. Find the value of the following:

(*i*) 7^4 (*ii*) $(-3)^5$ (*iii*) $\left(\frac{2}{3}\right)^3$ (*iv*) $\left(-\frac{4}{5}\right)^4$

Solution.

(*i*) $7^4 = 7 \times 7 \times 7 \times 7 = 2401$.

(*ii*) $(-3)^5 = (-3) \times (-3) \times (-3) \times (-3) \times (-3) = -243$.

(*iii*) $\left(\frac{2}{3}\right)^3 = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{2 \times 2 \times 2}{3 \times 3 \times 3} = \frac{8}{27}$.

(*iv*) $\left(-\frac{4}{5}\right)^4 = \left(-\frac{4}{5}\right) \times \left(-\frac{4}{5}\right) \times \left(-\frac{4}{5}\right) \times \left(-\frac{4}{5}\right) = \frac{256}{625}$.

Example 3. Simplify the following:

(*i*) 3×10^3 (*ii*) $2^6 \times 7$ (*iii*) $2^3 \times (-3)^2$

Solution.

(*i*) $3 \times 10^3 = 3 \times 10 \times 10 \times 10 = 3000$.

(*ii*) $2^6 \times 7 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 7 = 448$.

(*iii*) $2^3 \times (-3)^2 = 2 \times 2 \times 2 \times (-3) \times (-3) = 8 \times 9 = 72$.

Example 4. Which is the greater number 6^3 or 3^6?

Solution. $6^3 = 6 \times 6 \times 6 = 216$

and $3^6 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 27 \times 27 = 729$.

Since $729 > 216$, therefore, $3^6 > 6^3$.

Hence, the greater number is 3^6.

Powers of –1

$(-1)^2 = (-1) \times (-1) = 1$,

$(-1)^3 = (-1) \times (-1) \times (-1) = -1$,

$(-1)^4 = (-1) \times (-1) \times (-1) \times (-1) = 1$,

$(-1)^5 = (-1) \times (-1) \times (-1) \times (-1) \times (-1) = -1$ and so on.

Thus, we have

$(-1)^{\text{odd natural number}} = -1$ and $(-1)^{\text{even natural number}} = 1$.

Example 5. Evaluate the following:

(*i*) $(-1)^7$ (*ii*) $(-1)^{15}$ (*iii*) $(-1)^{100}$

Solution.

(*i*) As 7 is an odd natural number, $(-1)^7 = -1$.

(*ii*) As 15 is an odd natural number, $(-1)^{15} = -1$.

(*iii*) As 100 is an even natural number, $(-1)^{100} = 1$.

Example 6. Write the following numbers as powers of 2:

(*i*) 2 (*ii*) 32 (*iii*) 512

Solution.

(*i*) $2 = 2^1$

(*ii*) $32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$

(*iii*) $512 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^9$.

Example 7. Write the following numbers as powers of -2:

(*i*) -2 (*ii*) -8 (*iii*) 16 (*iv*) -32

Solution.

(*i*) $-2 = (-2)^1$

(*ii*) $-8 = (-2) \times (-2) \times (-2) = (-2)^3$

(*iii*) $16 = (-2) \times (-2) \times (-2) \times (-2) = (-2)^4$.

(*iv*) $-32 = (-2) \times (-2) \times (-2) \times (-2) \times (-2) = (-2)^5$.

Example 8. Express the following numbers in exponential form:

(*i*) 343 (*ii*) 729 (*iii*) 15625

Solution.

(*i*) $343 = 7 \times 7 \times 7 = 7^3$

(*ii*) $729 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6$

(*iii*) $15625 = 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^6$

Example 9. Find the value of x in each of the following:

(*i*) $3^x = 243$ (*ii*) $(-5)^x = -125$ (*iii*) $\left(-\frac{2}{3}\right)^x = \frac{16}{81}$.

Solution.

(*i*) $3^x = 243 \Rightarrow 3^x = 3 \times 3 \times 3 \times 3 \times 3$

$\Rightarrow 3^x = 3^5 \Rightarrow x = 5$.

(*ii*) $(-5)^x = -125 \Rightarrow (-5)^x = (-5) \times (-5) \times (-5)$

$\Rightarrow (-5)^x = (-5)^3 \Rightarrow x = 3$.

(*iii*) $\left(-\frac{2}{3}\right)^x = \frac{16}{81} \Rightarrow \left(-\frac{2}{3}\right)^x = \left(-\frac{2}{3}\right) \times \left(-\frac{2}{3}\right) \times \left(-\frac{2}{3}\right) \times \left(-\frac{2}{3}\right)$

$\Rightarrow \left(-\frac{2}{3}\right)^x = \left(-\frac{2}{3}\right)^4 \Rightarrow x = 4$.

Example 10. Write the prime factorisation of the following numbers in the exponential form:

(*i*) 48 (*ii*) 420 (*iii*) 2520.

Solution.

(*i*) $48 = 2 \times 2 \times 2 \times 2 \times 3$

$= 2^4 \times 3^1.$

(*ii*) $420 = 2 \times 2 \times 3 \times 5 \times 7$

$= 2^2 \times 3^1 \times 5^1 \times 7^1.$

(*iii*) $2520 = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7$

$= 2^3 \times 3^2 \times 5^1 \times 7^1.$

2	48
2	24
2	12
2	6
	3

2	420
2	210
3	105
5	35
	7

Exercise 4.1

1. Fill in the blanks:

(*i*) In the expression 3^7, base = … and exponent = …

(*ii*) In the expression $(-7)^5$, base = … and exponent = ….

(*iii*) In the expression $\left(\frac{2}{5}\right)^{11}$, base = … and exponent = ….

(*iv*) If base is 6 and exponent is 8, then exponential form = …

2. Find the value of the following:

(*i*) 2^6 (*ii*) 9^3 (*iii*) 5^5 (*iv*) $(-6)^4$

(*v*) $(-5)^5$ (*vi*) $\left(\frac{2}{3}\right)^4$ (*vii*) $\left(-\frac{2}{3}\right)^5$ (*viii*) $(-2)^9$.

3. Express the following in the exponential form:

(*i*) $6 \times 6 \times 6 \times 6$ (*ii*) $t \times t$ (*iii*) $b \times b \times b \times b$

(*iv*) $5 \times 5 \times 7 \times 7 \times 7$ (*v*) $2 \times 2 \times a \times a$ (*vi*) $a \times a \times a \times c \times c \times c \times c \times d$.

4. Simplify the following:

(*i*) 2×10^3 (*ii*) $2^5 \times 9$ (*iii*) $5^2 \times 3^3$ (*iv*) $3^2 \times 10^4$

5. Simplify the following:

(*i*) $(-3) \times (-2)^3$ (*ii*) $(-3)^2 \times (-5)^2$ (*iii*) $(-4)^3 \times 5^2$ (*iv*) $(-2)^3 \times (-10)^4$

(*v*) $(-1)^9$ (*vi*) $(-1)^{99}$ (*vii*) $(-1)^{122}$

6. Identify the greater number in each of the following:

(*i*) 4^3 or 3^4 (*ii*) 5^3 or 3^5 (*iii*) 2^8 or 8^2

(*iv*) 4^5 or 5^4 (*v*) 2^{10} or 10^2 (*vi*) 3.4×10^4 or 1.5×10^5

7. Write the following numbers as powers of 2:

(*i*) 8 (*ii*) 128 (*iii*) 1024

8. Write the following numbers as powers of 3:

(*i*) 27 (*ii*) 243 (*ii*) 2187

9. To what power (-2) should be raised to get 16?

10. Write the following numbers as powers of (-3):

(*i*) 9 (*ii*) -27 (*iii*) 81

11. Find the value of x in each of the following:

(*i*) $7^x = 343$ (*ii*) $3^x = 729$ (*iii*) $(-8)^x = -512$

(*iv*) $(-4)^x = -1024$ (*v*) $(-2)^x = 64$ (*vi*) $(-5)^x = 625$

(*vii*) $\left(\frac{2}{5}\right)^x = \frac{32}{3125}$ (*viii*) $\left(-\frac{3}{4}\right)^x = -\frac{243}{1024}$.

12. Write the prime factorization of the following numbers in the exponential form:

(*i*) 72 (*ii*) 360 (*iii*) 405 (*iv*) 648

(*v*) 540 (*vi*) 2280 (*vii*) 3600 (*viii*) 4725.