

# 3

## Playing with Numbers

### IN THIS CHAPTER, YOU WILL LEARN:

- Factors and multiples of a natural number
- Common factors and common multiples
- Types of natural numbers — even and odd numbers, prime and composite numbers, twin-primes, co-primes and perfect numbers
- Divisibility tests of natural numbers
- General properties of divisibility
- Prime factorisation of natural numbers
- H.C.F. of natural numbers
- L.C.M. of natural numbers
- Properties of H.C.F. and L.C.M. of natural numbers

### FACTORS AND MULTIPLES

When two or more (natural) numbers are multiplied together then the result is called their **product**.

*For example:*

- (i) The product of 3 and 5 is 15 i.e.  $3 \times 5 = 15$ .
- (ii) The product of 2, 3 and 7 is 42 i.e.  $2 \times 3 \times 7 = 42$ .

Each of the numbers which are multiplied together is called a **factor** of that product, and the product is called a **multiple** of each number.

*For example:*

As  $3 \times 5 = 15$ , 3 is a factor of 15 and 15 is a multiple of 3, also 5 is a factor of 15 and 15 is a multiple of 5.

Thus, a (natural) number which divides a given (natural) number exactly is called a **factor** (or **divisor**) of that given number, and the given number is called a **multiple** of that number.

In other words, if  $a$ ,  $b$  and  $c$  are natural numbers such that  $ab = c$ , then each of  $a$  and  $b$  is a factor (or divisor) of  $c$  and  $c$  is called a multiple of each of  $a$  and  $b$ .

Hence,  $a$  is a factor of  $c$  if on dividing  $c$  by  $a$ , the remainder is zero.

*For example:*

- (i) As  $3 \times 5 = 15$  and  $1 \times 15 = 15$ , 1, 3, 5 and 15 are all factors of 15 and 15 is a multiple of each of 1, 3, 5 and 15.
- (ii)  $3 \times 4 = 12$ ,  $2 \times 6 = 12$  and  $1 \times 12 = 12$ , 1, 2, 3, 4, 6 and 12 are all factors of 12 and 12 is multiple of each of 1, 2, 3, 4, 6 and 12.

Similarly:

The factors of 18 are: 1, 2, 3, 6, 9, 18.

The factors of 42 are: 1, 2, 3, 6, 7, 14, 21, 42.

The multiples of 3 are: 3, 6, 9, 12, 15, ....

The multiples of 4 are: 4, 8, 12, 16, 20, ...

The multiples of 14 are: 14, 28, 42, 56, 70, ...

From the above examples, we notice that:

- (i) 1 is a factor of every number.
- (ii) the only factor of 1 is 1 itself.
- (iii) every number is a factor of itself.
- (iv) every number (other than 1) has atleast two factors, namely 1 and itself.
- (v) every factor of a number is less than or equal to that number.
- (vi) every number has a finite number of factors.
- (vii) every number is a multiple of itself.
- (viii) every multiple of a number is greater than or equal to the number itself.
- (ix) every number has an infinite number of its multiples.

## Common factors and common multiples

Consider the following examples:

(i) The factors of 12 are ①, ②, ③, 4, ⑥ and 12.

The factors of 18 are ①, ②, ③, ⑥, 9 and 18.

Note that the numbers 1, 2, 3 and 6 are factors of both 12 and 18.

They are the **common factors** of 12 and 18.

(ii) The factors of 8 are ①, 2, 4, and 8.

The factors of 15 are ①, 3, 5 and 15.

Note that 1 is the only common factor of 8 and 15.

(iii) The multiples of 4 are 4, 8, ⑫, 16, 20, ⑲, 28, 32, ⑳, .....

The multiples of 6 are 6, ⑫, 18, ⑲, 30, ⑳, 42, ....

Note that the numbers 12, 24, 36, ... are multiples of both 4 and 6.

They are the **common multiples** of 4 and 6.

## TYPES OF NATURAL NUMBERS

### Even number

A natural number which is exactly divisible by 2 is called an **even natural number**.

Even natural numbers are: 2, 4, 6, 8, 10, 12, 14, ...

Note that there are infinitely many even natural numbers.

Even numbers  
end in 0, 2,  
4, 6, 8

### Odd number

A natural number which is not exactly divisible by 2 is called an **odd natural number**.

Odd natural numbers are: 1, 3, 5, 7, 9, 11, 13, ...

Note that there are infinitely many odd natural numbers.

Odd numbers  
end in 1, 3,  
5, 7, 9

## Prime number

A natural number greater than 1 is called a **prime number** if its only factors are 1 and the number itself.

Some prime numbers are: 2, 3, 5, 7, 11, 13, 17, ...

## Composite number

A natural number is called a **composite number** if it has more than two different factors.

Some composite numbers are: 4, 6, 8, 9, 10, 12, 14, 15, ...

## Sieve of Eratosthenes

How can we enlist all the prime numbers from 1 to 100?

Eratosthenes (274 BC–194 BC), a Greek mathematician, gave the following simple method to separate the prime numbers:

*Step 1.* Write all the natural numbers from 1 to 100 in the rows of 10.

1	2	3	<del>4</del>	5	<del>6</del>	7	<del>8</del>	<del>9</del>	<del>10</del>
11	<del>12</del>	13	<del>14</del>	<del>15</del>	<del>16</del>	17	<del>18</del>	19	<del>20</del>
<del>21</del>	<del>22</del>	23	<del>24</del>	<del>25</del>	<del>26</del>	<del>27</del>	<del>28</del>	29	<del>30</del>
31	<del>32</del>	<del>33</del>	<del>34</del>	<del>35</del>	<del>36</del>	37	<del>38</del>	<del>39</del>	<del>40</del>
41	<del>42</del>	43	<del>44</del>	<del>45</del>	<del>46</del>	47	<del>48</del>	<del>49</del>	<del>50</del>
<del>51</del>	<del>52</del>	53	<del>54</del>	<del>55</del>	<del>56</del>	<del>57</del>	<del>58</del>	59	<del>60</del>
61	<del>62</del>	<del>63</del>	<del>64</del>	<del>65</del>	<del>66</del>	67	<del>68</del>	<del>69</del>	<del>70</del>
71	<del>72</del>	73	<del>74</del>	<del>75</del>	<del>76</del>	<del>77</del>	<del>78</del>	79	<del>80</del>
<del>81</del>	<del>82</del>	83	<del>84</del>	<del>85</del>	<del>86</del>	<del>87</del>	<del>88</del>	89	<del>90</del>
<del>91</del>	<del>92</del>	<del>93</del>	<del>94</del>	<del>95</del>	<del>96</del>	97	<del>98</del>	<del>99</del>	<del>100</del>

*Step 2.* As 1 is neither prime nor composite, separate it by making a box around it.

*Step 3.* Begin with 2 which is prime. Encircle it and cross out all the multiples of 2.

*Step 4.* The next uncrossed number 3 is prime. Encircle it and cross out all the multiples of 3. Notice that some of the numbers have already been crossed.

*Step 5.* The next uncrossed number 5 is prime. Encircle it and cross out all the multiples of 5.

*Step 6.* Continue the above process. Encircling the prime numbers and crossing out their multiples until we cannot cross out any more numbers.

Thus the prime numbers from 1 to 100 are

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

Perhaps Eratosthenes made holes in the paper instead of crossing out the numbers and the paper must have looked like a sieve and hence the above method came to be known as **sieve method**.

## Twin-primes

Pairs of prime numbers whose difference is 2 are called **twin-primes**.

Pairs of twin-primes between 1 and 100 are:

3, 5; 5, 7; 11, 13; 17, 19; 29, 31; 41, 43; 59, 61 and 71, 73.

## Prime-triplet

A triplet of consecutive prime numbers which differ by 2 is called a prime-triplet.

The only prime-triplet is 3, 5, 7.

## Co-prime numbers

Two natural numbers are called **co-prime numbers** if they have no common factor other than 1.

Some examples of co-prime numbers are: 2, 3; 3, 8; 5, 6; 8, 15.

## Perfect number

If the sum of all the factors of a number is equal to twice the number, then the number is called a perfect number.

**For example:**

(i) The factors of 6 are 1, 2, 3, and 6. Note that

$$1 + 2 + 3 + 6 = 12 = 2 \times 6, \text{ so } 6 \text{ is a perfect number.}$$

(ii) The factors for 28 are 1, 2, 4, 7, 14 and 28. Note that

$$1 + 2 + 4 + 7 + 14 + 28 = 56 = 2 \times 28, \text{ so } 28 \text{ is a perfect number.}$$

## Remarks

- 1 is neither prime nor composite.
- Every natural number (except 1) is either prime or composite.
- 2 is only prime number which is even. All other prime numbers are odd. Note that every odd number is not prime, for example, 9 is an odd number but not a prime number.
- There are infinitely many prime numbers.
- Two prime numbers are always co-prime.
- Two co-prime numbers may not be prime.

■ **Example 1.** Write all the factors of (i) 24 (ii) 36

**Solution.**

(i) The factors of 24 are: 1, 2, 3, 4, 6, 8, 12, 24

(ii) The factors of 36 are: 1, 2, 3, 4, 6, 9, 12, 18, 36.

■ **Example 2.** Write all the multiples of 9 upto 100.

**Solution.** The multiples of 9 upto 100 are: 9, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99.

■ **Example 3.** Find the common factors of 60, 75 and 105.

**Solution.** The factors of 60 are (1), 2, (3), 4, (5), 6, 10, 12, (15), 30 and 60.

The factors of 75 are (1), (3), (5), (15), 25, and 75.

The factors of 105 are (1), (3), (5), 7, (15), 21, 35 and 105.

The common factors of 60, 70 and 105 are 1, 3, 5 and 15.

■ **Example 4.** Find the common multiples of 4, 6 and 8.

**Solution.** The multiples of 4 are 4, 8, 12, 16, 20, (24), 28, 32, 36, 40, 44, (48), 52, 56, 60, 64, 68, (72), ...

The multiples of 6 are 6, 12, 18, (24), 30, 36, 42, (48), 54, 60, (72), ...

The multiples of 8 are 8, 16, (24), 32, 40, (48), 56, 64, (72), ...

The common multiples of 4, 6 and 8 are 24, 48, 72, ...

■ **Example 5.** Write the even numbers between 13 and 24.

**Solution.** The even numbers between 13 and 24 are: 14, 16, 18, 20, 22.

■ **Example 6.** Write the odd numbers between 121 and 136.

**Solution.** The odd numbers between 121 and 136 are: 123, 125, 127, 129, 131, 133, 135.

■ **Example 7.** Write three consecutive even whole numbers succeeding 78.

**Solution.** Three consecutive even whole numbers succeeding 78 are: 80, 82, 84.

■ **Example 8.** Express each of the following even numbers as the sum of two odd prime numbers:

- (i) 18                      (ii) 44

**Solution.**

(i)  $18 = 7 + 11$

(Note that this could be one of the ways. There can be other ways also. Observe that:  
 $18 = 5 + 13$ )

(ii)  $44 = 13 + 31$

(Note that:  $44 = 7 + 37$ ;  $44 = 3 + 41$ )

■ **Example 9.** Express each of the following numbers as the sum of three odd prime numbers:

- (i) 31                      (ii) 61

**Solution.**

(i)  $31 = 3 + 5 + 23$

(Note that this could be one of the ways. There can be other ways also. Observe that:  
 $31 = 5 + 7 + 19$ )

(ii)  $61 = 7 + 13 + 41$

(Note that:  $61 = 13 + 17 + 31$ )

### Exercise 3.1

1. Fill in the blanks:

- (i) A number having exactly two factors is called a ....
- (ii) A number having more than two factors is called a ...
- (iii) 1 is neither ..... nor .....
- (iv) The smallest prime number is ....
- (v) The smallest odd prime number is ....
- (vi) The smallest composite number is .....
- (vii) The smallest odd composite number is ...
- (viii) All prime numbers (except 2) are ....

2. State where the following statements are true (T) or false (F):

- (i) The sum of three odd numbers is an even number.
- (ii) The sum of two odd numbers and one even number is an even number.
- (iii) The product of two even numbers is always an even number.
- (iv) The product of three odd numbers is an odd number.
- (v) If an even number is divided by 2, the quotient is always an odd number.
- (vi) All prime numbers are odd.
- (vii) All even numbers are composite.

- (viii) Prime numbers do not have any factors.  
(ix) A natural number is called a composite number if it has atleast one more factor other than 1 and the number itself.  
(x) Two consecutive numbers cannot be both prime.  
(xi) Two prime numbers are always co-prime numbers.
3. Write all the factors of the following natural numbers:  
(i) 68      (ii) 27      (iii) 210.
4. Write first six multiples of the following natural numbers:  
(i) 3      (ii) 5      (iii) 12.
5. Match the items in column 1 with the items in column 2:
- | <b>Column 1</b> | <b>Column 2</b>    |
|-----------------|--------------------|
| (i) 15          | (a) Multiple of 8  |
| (ii) 36         | (b) Factor of 30   |
| (iii) 16        | (c) Multiple of 70 |
| (iv) 20         | (d) Factor of 50   |
| (v) 25          | (e) Multiple of 9  |
| (vi) 210        | (f) Factor of 20   |
6. Find the common factors of:  
(i) 20 and 28      (ii) 35 and 50      (iii) 56 and 120
7. Find the common factors of:  
(i) 4, 8, 12      (ii) 10, 30 and 45.
8. Write all natural numbers less than 100 which are common multiples of 3 and 4.
9. (i) Write the odd numbers between 36 and 53.  
(ii) Write the even numbers between 232 and 251.
10. (i) Write four consecutive odd numbers succeeding 79.  
(ii) Write three consecutive even numbers preceding 124.
11. What is greatest prime number between 1 and 10?
12. Which of the following numbers are prime?  
(i) 23      (ii) 51      (iii) 37      (iv) 61
13. The numbers 37 and 73 are prime numbers. Both these numbers have the same digits 3 and 7. Find such pairs of prime numbers upto 100.
14. Write seven consecutive composite numbers between 1 and 100 so that there is no prime number between them.
15. What are twin-prime numbers? Write all pairs of twin-prime numbers between 40 and 100.
16. Which of the following pairs of numbers are co-prime?  
(i) 18 and 35      (ii) 15 and 37      (iii) 81 and 16  
(iv) 17 and 68      (v) 215 and 216      (vi) 30 and 415.
17. Express each of the following numbers as the sum of two odd primes:  
(i) 24      (ii) 36      (iii) 84      (iv) 98
18. Express each of the following numbers as the sum of twin-primes:  
(i) 24      (ii) 36      (iii) 84      (iv) 120
19. Express each of the following numbers as the sum of three odd primes:  
(i) 21      (ii) 35      (iii) 53      (iv) 63
20. Write five pairs of prime numbers less than 20 whose sum is divisible by 5.

## DIVISIBILITY TESTS OF NATURAL NUMBERS

So far the only method we know to test whether or not a number is divisible by another number is to perform actual division and see whether or not the remainder is zero. This is time consuming. However, there are certain divisibility tests to check (without actual division) whether or not a number is divisible by a certain given number.

### Divisibility by 10

*A number is divisible by 10 if its last (unit's) digit is 0.*

*For example:*

Each of the numbers 230, 1580 and 23740 is divisible by 10 whereas none of the numbers 239, 1576 and 23458 is divisible by 10.

### Divisibility by 5

*A number is divisible by 5 if its last (unit's) digit is 0 or 5.*

*For example:*

Each of the numbers 230, 475, 1020 and 23485 is divisible by 5 whereas none of the numbers 237, 1248 and 13786 is divisible by 5.

### Divisibility by 2

*A number is divisible by 2 if its last (unit's) digit is divisible by 2 i.e. if its last (unit's) digit is 0, 2, 4, 6 or 8.*

*For example:*

Each of the numbers 120, 372, 2584, 3786 and 45988 is divisible by 2 whereas none of the number 121, 233, 2575, 37777 and 38359 is divisible by 2.

### Divisibility by 4

*A number is divisible by 4 if the number formed by its last two (unit's and ten's) digits is divisible by 4.*

■ **Example 1.** Which of the following numbers are divisible by 4?

- (i) 350736      (ii) 7385082.

**Solution.**

- (i) The number formed by the last two digits is 36.

Since 36 is divisible by 4, the given number 350736 is divisible by 4.

- (ii) The number formed by the last two digits is 82.

As 82 is not divisible by 4, the given number 7385082 is not divisible by 4.

### Divisibility by 8

*A number is divisible by 8 if the number formed by its last three (unit's, ten's and hundred's) digits is divisible by 8.*

■ **Example 2.** Which of the following numbers are divisible by 8?

- (i) 78056      (ii) 394148.

**Solution.**

- (i) The number formed by the last three digits is 056 i.e. 56.

Since 56 is divisible by 8, the number 78056 is divisible by 8.