4 Integers

In this chapter, you will learn:

- Integers
- Representation of integers on number line
- Absolute value of an integer
- Comparison of integers
- Use of integers
- Addition of integers using number line
- Addition of integers
- Properties of addition of integers
- Subtraction of integers using number line
- Successor and predecessor of an integer
- Addition/subtraction of three or more integers.

INTEGERS

We know that on adding any two whole numbers, we always get a whole number. Is this true for subtraction as well? Let us examine it.

Consider the following examples:

$$37 - 25 = 12$$
$$37 - 37 = 0$$
$$25 - 37 = ?$$

We observe that in the last case, there is no answer in the system of whole numbers *i.e.* when a bigger whole number is subtracted from a smaller whole number, we do not get a whole number; thus the system of whole numbers is inadequate for subtraction. Therefore, there is a need to enlarge the system of whole numbers to provide an answer to all questions of subtraction. We introduce negative numbers $-1, -2, -3, -4, -5$, ... which are opposite to natural numbers 1, 2, 3, 4, 5, … .

These new numbers together with whole numbers are called **integers.**

Thus, the numbers ….., $-4, -3, -2, -1$, 0, 1, 2, 3, 4, ….. are **integers.**

The natural numbers 1, 2, 3, 4, ….. are called **positive integers.**

The natural numbers $-1, -2, -3, -4$, ….. are called **negative integers.**

The number 0 is an integer. It is neither positive nor negative.

Positive integers are also written as +1, +2, +3, …..,

However, the plus sign (+) is usually omitted.

Representation of integers on number line

To represent integers on a line, we proceed as under:

(1) Draw a straight line, mark a point on it and label it as 0 (zero).

(2) Select a unit of length. Mark points on the line at unit length intervals from each other on both sides of 0 (zero).

(3) Label the points to the right of 0 (zero) successively as 1, 2, 3,... and the points to the left of 0 (zero) successively as $-1, -2, -3,...$

(4) Put arrows on each side of the line to show that the line continues indefinitely in both directions.

Thus, every integer can be represented by some point on the line. The line drawn above is called the **number line**.

From the number line, we observe that:

1. *There is no largest integer and no smallest integer.*

2. *An integer is greater than all those integers that lie to its left on the number line.*

In other words, an integer is greater than the other integer if the first integer lies on the right of the second integer on the number line.

For example:

As 5 lies on the right of -2 on the number line, so $5 > -2$.

Similarly, $0 > -3$ and $-1 > -4$.

3. *An integer is less than all those integers that lie to its right on the number line.*

In other words, an integer is less than the other integer if the first integer lies on the left of the second integer on the number line.

For example:

As -2 lies on the left of 5 on the number line, so $-2 < 5$.

Similarly, $-3 < 0$ and $-4 < -1$.

From the above observations it follows that:

(*i*) *Zero is less than every positive integer.*

(*ii*) *Zero is greater than every negative integer.*

(*iii*) *Every positive integer is greater than every negative integer.*

(*iv*) *Farther a number from zero on the right, larger is its value.*

(*v*) *Farther a number from zero on the left, smaller is its value.*

Absolute value of an integer

The **absolute value** of an integer is its numerical value regardless of its sign.

The absolute value of an integer a is written as $|a|$.

Thus, $|5| = 5$, $|13| = 13$, $|0| = 0$, $|-3| = 3$, $|-7| = 7$ etc.

Note that the absolute value of an integer is always non-negative.

If a is an integer, then

(*i*) $|a| = a$ if $a \geq 0$ *i.e.* if a is positive or zero.

(*ii*) $|a| = -a$ if $a < 0$ *i.e.* if a is negative.

Comparison of integers

The rules for the comparison of integers are:

1. Every positive integer is greater than zero and every negative integer is less than zero.

2. Every positive integer is greater than every negative integer; *alternatively*, every negative integer is less than every positive integer.

3. Given two positive integers – compare them as whole numbers.

4. Given two negative integers – the integer with smaller absolute value is greater; *alternatively*, the integer with bigger absolute value is smaller.

For example:

(*i*) $-3 > -7$ because $|-3| < |-7|$ *i.e.* $3 < 7$.

(*ii*) $-25 > -39$ because $|-25| < |-39|$ *i.e.* $25 < 39$.

(*iii*) $-78 < -43$ because $|-78| > |-43|$ *i.e.* $78 > 43$.

Uses of integers

In our daily life, we come across many statements which are opposite to each other. Integers are used to express these statements in mathematical terms.

For example:

(*i*) Profits are represented by positive integers and losses are represented by negative integers.

(*ii*) Deposits in a bank (or post office etc.) are represented by positive integers and withdrawals are represented by negative integers.

(*iii*) Heights above sea level are represented by positive integers and depths below sea level are represented by negative integers.

(*iv*) Temperatures above freezing point are represented by positive integers and below freezing point are represented by negative integers and so on.

Thus, a gain of ₹ 500 is represented by + ₹ 500 (or ₹ 500) and a loss of ₹ 500 is represented by – ₹ 500.

Example 1. Write opposite of the following:

(*i*) Decrease in weight (*ii*) Profit of ₹ 7500

(*iii*) Walking 800 m towards north (*iv*) 70 m below sea level.

Solution.

(*i*) Increase in weight (*ii*) Loss of ₹ 7500

(*iii*) Walking 800 m towards south (*iv*) 70 m above sea level.

Example 2. Write each of the following using appropriate sign '+' or '–':

(*i*) A withdrawal of ₹ 2000.

(*ii*) An aeroplane is flying at a height of 3500 m above sea level.

(*iii*) 5°C below 0°C.

Solution.

(*i*) – ₹ 2000 (*ii*) + 3500 m (*iii*) – 5°C.

Example 3. Draw a number line and answer the following questions:

(*i*) Which integers lie between – 6 and – 1?

(*ii*) Which is the largest integer among them?

(*iii*) Which is the smallest integer among them?

Solution. The number line is shown below:

(*i*) The integers between – 6 and – 1 are – 5, – 4, – 3, – 2.

(*ii*) The largest integer among these is -2.

(*iii*) The smallest integer among these is -5.

Example 4. Draw a number line and answer the following questions:

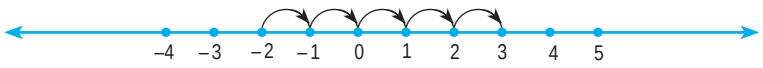
(*i*) Which number will we reach if we move 5 units to the right of -2?

(*ii*) Which number will we reach if we move 6 units to the left of 1?

(*iii*) In which direction should we move to reach -7 from -2?

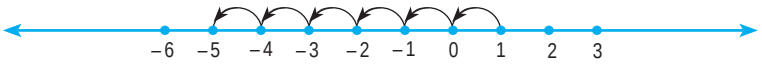
Solution. Draw the number line.

(*i*)



After moving 5 units to the right of -2, we reach at 3.

(*ii*)



After moving 6 units to the left of 1, we reach at -5.

(*iii*)

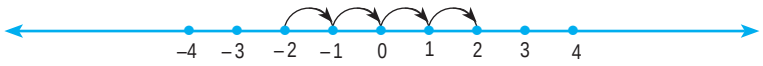
To reach -7 from -2, we have to move in the left direction,

Example 5. Using the number line, write the integer which is:

(*i*) 4 more than -2 (*ii*) 6 less than 1 (*iii*) 5 less than -1.

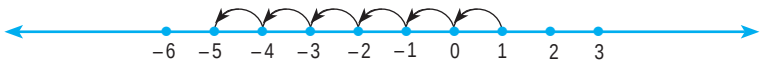
Solution.

(*i*) Since we want to find an integer which is 4 more than -2, so we start from -2 and move 4 units to the right.



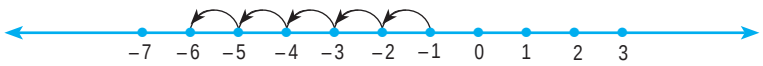
We reach at 2, so the required integer is 2.

(*ii*) Since we want to find an integer which is 6 less than 1, so we start from 1 and move 6 units to the left.



We reach at -5, so the required integer is -5.

(*iii*) Since we want to find an integer which is 5 less than -1, so we start from -1 and move 5 units to the left.



We reach at -6, so the required integer is -6.

Example 6. Evaluate the following:

(*i*) $|-7| + |10|$ (*ii*) $|-12| + |-9|$ (*iii*) $|-8| - |6|$ (*iv*) $|-15| - |-11|$.

Solution.

(*i*) Since $|-7| = 7$ and $|10| = 10$,

$\therefore\ |-7| + |10| = 7 + 10 = 17.$

(*ii*) Since $|-12| = 12$ and $|-9| = 9$,

$\therefore\ |-12| + |-9| = 12 + 9 = 21.$

(*iii*) Since $|-8| = 8$ and $|6| = 6$,

$\therefore\ |-8| - |6| = 8 - 6 = 2.$

(*iv*) Since $|-15| = 15$ and $|-11| = 11$

$\therefore\ |-15| - |-11| = 15 - 11 = 4.$

Example 7. Use the appropriate symbol < or > to fill in the following blanks:

(*i*) $-5 \ldots 3$ (*ii*) $5 \ldots -1$ (*iii*) $0 \ldots -6$

(*iv*) $-10 \ldots 10$ (*v*) $-5 \ldots -3$ (*vi*) $-64 \ldots -203$.

Solution.

(*i*) As every negative integer is less than every positive integer, $-5 < 3$.

(*ii*) As every positive integer is greater than every negative integer, $5 > -1$.

(*iii*) As zero is greater than every negative integer, $0 > -6$.

(*iv*) As every negative integer is less than positive integer, $-10 < 10$.

(*v*) As $|-5| > |-3|$ *i.e.* $5 > 3$, $-5 < -3$.

(*vi*) As $|-64| < |-203|$ *i.e.* $64 < 203$, $-64 > -203$.

Example 8. Arrange the following integers in ascending order:

$-33, 37, 5, 615, -9.$

Solution. For negative integers $-33, -9$,

we have $-33 < -9$ because $|-33| > |-9|$ *i.e.* $33 > 9$.

For positive integers 37, 5, 615, we have $5 < 37 < 615$.

As every negative integer is less than every positive integer, therefore, we get

$-33 < -9 < 5 < 37 < 615.$

Hence, the given integers in ascending order are:

$-33, -9, 5, 37, 615$

Ascending means smaller to greater

Example 9. Arrange the following integers in descending order:

$189, 2056, -49, -6, -11, 678.$

Solution. For positive integers 189, 2056, 678,

we have $2056 > 678 > 189$.

For negative integers $-49, -6, -11$,

we have $-6 > -11 > -49$

because $|-6| < |-11| < |-49|$ *i.e.* $6 < 11 < 49$.

As every positive integer is greater than every negative integer, therefore, we get

$2056 > 678 > 189 > -6 > -11 > -49.$

Hence, the given integers in descending order are:

$2056, 678, 189, -6, -11, -49.$

Descending means greater to smaller

Exercise 4.1

1. Write the opposite of the following:

(*i*) Loss of ₹ 5000 (*ii*) 30 km East of Delhi

(*iii*) 200 m above sea level (*iv*) 325 BC

(*v*) Spending ₹ 2700 (*vi*) 25°C above freezing point.

2. Write each of the following using appropriate sign '+' or '–':
(*i*) Gain of 3 kg weight (*ii*) Earning ₹ 1340
(*iii*) 20°C below freezing point (*iv*) Loss of ₹ 470
(*v*) Depositing ₹ 2500 in a bank (*vi*) 240 m below sea level
(*vii*) A jet plane flying at a height of 9320 m.
(*viii*) 6 m down in the basement of a building.

3. In each of following pairs, which number is to the right of the other on the number line?
(*i*) 3, 5 (*ii*) 0, –2 (*iii*) –3, –5 (*iv*) 2, –7.

4. In each of the following pairs, which number is to the left of the other on the number line?
(*i*) –3, 0 (*ii*) 2, –5 (*iii*) –4, –7 (*iv*) –10, –16.

5. Draw a number line and answer the following questions:
(*i*) Which integers lie between –9 and –2?
(*ii*) Which is the largest among them?
(*iii*) Which is the smallest among them?

6. Write four consecutive integers just greater than –9.

7. Write four consecutive integers just before –2.

8. Write all integers in the increasing order between the given pairs:
(*i*) 0 and –6 (*ii*) –8 and –15 (*iii*) –30 and –23.

9. Draw a number line and answer the following questions:
(*i*) Which number will we reach if we move 6 units to the right of –1?
(*ii*) Which number will we reach if we move 7 units to the left of 2?
(*iii*) In which direction should we move to reach 3 from –3?
(*iv*) In which direction should we move to reach –8 from –3?

10. Using the number line, write the integer which is:
(*i*) 5 more than –1 (*ii*) 5 less than –1
(*iii*) 7 less than 2 (*iv*) 3 more than –7.

11. Evaluate the following:
(*i*) |13 – 5| (*ii*) |5 – 13| (*iii*) |–11| + |9|
(*iv*) |–8| + |–6| (*v*) |7| – |–3| (*vi*) |–19| – |–13|.

12. Evaluate the following:
(*i*) |–3| + |7| + |0| (*ii*) |–6| – |7| + |–12|
(*iii*) –|–5| + |–6| – |–1| (*iv*) |–9| – |14| + |–18|.

13. Use the appropriate symbol < or > to fill in the following blanks:
(*i*) –3 ….. 7 (*ii*) 0 ….. –2 (*iii*) –10 ….. –11
(*iv*) –6 …… –2 (*v*) –5 ….. –13 (*vi*) –30 ….. –19.

14. Arrange the following integers in ascending order:
(*i*) –5, 3, 0, –9, 2 (*ii*) –28, –33, 9, –4, –31, –2, 35

15. Arrange the following integers in descending order:
(*i*) –31, 25, –37, 43, 0, –5
(*ii*) –101, 95, –3, –8, 36, –7, –84.

16. State whether the following statements are true (T) or false (F):
(*i*) 0 is the smallest positive integer.
(*ii*) Every negative integer is less than every natural number.
(*iii*) –7 is to the right of –6 on the number line.
(*iv*) The absolute value of an integer is always greater than the integer.