

8

Decimals

IN THIS CHAPTER, YOU WILL LEARN:

- Decimal numbers
- Representation of decimals on a number line
- Conversion of decimal numbers to fractions
- Conversion of fractions to decimal numbers
- Comparison of decimal numbers
- Addition/subtraction of decimal numbers
- Multiplication/division of a decimal number by 10, 100, 1000,
- Uses of decimal numbers in real life

DECIMAL NUMBER

Look at the number 7326. The position of each digit determines the value of that digit. Counting from right, the first digit '6' is the units digit and it represents six units. The second digit '2' is the tens digit and it represents two tens *i.e.* twenty. The next digit '3' is the hundreds digit and it represents 3 hundreds and '7' represents seven thousands.

So 7326 can be written as

$$\begin{aligned}7326 &= 7 \times 1000 + 3 \times 100 + 2 \times 10 + 6 \times 1 \\ &= 7 \text{ thousands} + 3 \text{ hundreds} + 2 \text{ tens} + 6 \text{ ones}\end{aligned}$$

In the place value chart, we have

Places	Thousands	Hundreds	Tens	Ones
Value	1000	100	10	1
Number	7	3	2	6

Thus, as we move from right to left, the place value of each digit is multiplied by 10. In the above number, counting from left, the first digit is thousands digit, the second digit is hundreds digit, the next one is tens digit and the last digit is the ones digit *i.e.* as we move from left to right, the place value of each digit is divided by 10.

If we extend this system to the right of ones place, the place of the digit next on the right of ones digit is the tenths digit (divide by 10) *i.e.* $\frac{1}{10}$ of the digit while the place value of the next digit is hundredths (divide by 100) *i.e.* $\frac{1}{100}$ of the digit and so on. Thus the place values of the digits that are on the left of ones digit in order are multiplied by 10, 100, 1000 and so on, and the place

values of the digits that are on the right of ones digit in order are divided by 10, 100, 1000 and so on.

To separate these, we put a point (small dot) after the ones digit. This point is called a **decimal point**.

For example:

- (i) The number 23.7 is read as twenty three point seven or twenty three and seven-tenths. It can be written as

$$\begin{aligned} 23.7 &= 2 \times 10 + 3 \times 1 + 7 \times \frac{1}{10} \\ &= 2 \text{ tens} + 3 \text{ ones} + 7 \text{ tenths} \end{aligned}$$

- (ii) The number 7.25 is read as seven point two five (not seven point twenty five). It can be written as

$$\begin{aligned} 7.25 &= 7 \times 1 + 2 \times \frac{1}{10} + 5 \times \frac{1}{100} \\ &= 7 \text{ units} + 2 \text{ tenths} + 5 \text{ hundredths} \end{aligned}$$

- (iii) The number 39.458 is read as thirty nine point four five eight. It can be written as

$$\begin{aligned} 39.458 &= 3 \times 10 + 9 \times 1 + 4 \times \frac{1}{10} + 5 \times \frac{1}{100} + 8 \times \frac{1}{1000} \\ &= 3 \text{ tens} + 9 \text{ units} + 4 \text{ tenths} + 5 \text{ hundredths} + 8 \text{ thousandths} \end{aligned}$$

The first position after the decimal point represents the tenths and is called the first decimal place. The second position after the decimal point represents the hundredths and is called the second decimal place. The third position after the decimal point represents thousandths and is called the third decimal place and so on.

Place value chart

PLACES	Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
VALUE		1000	100	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	

From the above examples and the place value chart, it follows that we can write $\frac{1}{10}$ as 0.1, $\frac{1}{100}$ as 0.01 and $\frac{1}{1000}$ as 0.001 and so on.

The numbers 23.7, 7.25 and 39.458 are all *decimal numbers*.

A decimal number has two parts — one part to the left of decimal point and the other to the right of decimal point.

The part to the left of decimal point is called *whole number (or integral) part* and the part to the right of decimal point is called *decimal part*.

For example, in the number 23.7, the whole number part = 23 and the decimal part is = .7

The number of digits in the decimal part of decimal number is the number of decimal places in it. For example, 23.7 has one decimal place, 7.25 has two decimal places and 39.458 has three decimal places.

Tenths

To understand *tenths*, look at the adjoining figure. The whole figure is divided into ten equal parts and one part has been shaded. The shaded part represents one-tenth of the whole figure *i.e.* $\frac{1}{10}$, it is also written as 0.1.



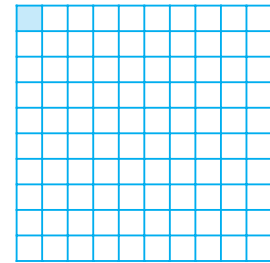
In the adjoining figure, the whole figure is divided into ten equal parts and three parts has been shaded. The shaded part represents three-tenths of the whole figure *i.e.* $\frac{3}{10}$, it is also written as 0.3.



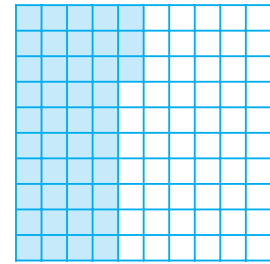
Similarly, $\frac{2}{10}$, $\frac{4}{10}$, $\frac{5}{10}$ are called 2-tenths, 4-tenths, 5-tenths etc.

Hundredths

To understand *hundredths*, look at the adjoining figure. The whole figure is divided into 100 equal parts and one part has been shaded. The shaded part represents one-hundredth of the whole figure *i.e.* $\frac{1}{100}$, it is also written as 0.01.



In the adjoining figure, the whole figure is divided into 100 equal parts and 43 parts has been shaded. The shaded part represents forty three-hundredths *i.e.* $\frac{43}{100}$, it is written as 0.43



Similarly, $\frac{7}{100}$, $\frac{29}{100}$ and $\frac{79}{100}$ are written as 0.07, 0.29 and 0.79 respectively etc.

Thousandths

If an object is divided into 1000 equal parts, each part is called one-thousandth of the whole object.

Thus, one-thousandth = $\frac{1}{1000}$, it is written as 0.001. If we take 15 parts out of 1000 equal parts, then 15 parts make $\frac{15}{1000}$ of the whole object, it is written as 0.015 etc.

Representation of decimal numbers on a number line

We know how to represent fractions on a number line. Let us now learn the representation of decimal numbers on a number line.

Let us represent 0.7 and 1.3 on a number line.

We know that 0.7 is more than 0 but less than 1. There are 7 tenths in 0.7. Divide the unit length between 0 and 1 into 10 equal parts and take 7 parts as shown. The point P represents 0.7



To represent 1.3, we know that it is more than 1 but less than 2. Divide the unit length between 1 and 2 into 10 equal parts and take 3 equal parts between 1 and 2 as shown. The point Q represents 1.3

■ **Example 1.** Write each of the following decimal numbers in the place value table:

(i) 20.6

(ii) 148.32

(iii) 5203.075

(iv) 37.008

Solution. The given decimal numbers are written in the place value table as under:

Places	...	Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths	...
Values		1000	100	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	
(i) 20.6				2	0	6			
(ii) 148.32			1	4	8	3	2		
(iii) 5203.075		5	2	0	3	0	7	5	
(iv) 37.008				3	7	0	0	8	

■ **Example 2.** Write the following decimal numbers in expanded form:

(i) 267.8

(ii) 32.05

(iii) 803.207

(iv) 7.008

Solution.

$$(i) 267.8 = 2 \times 100 + 6 \times 10 + 7 \times 1 + 8 \times \frac{1}{10}$$

$$= 200 + 60 + 7 + \frac{8}{10}$$

$$(ii) 32.05 = 3 \times 10 + 2 \times 1 + 0 \times \frac{1}{10} + 5 \times \frac{1}{100}$$

$$= 30 + 2 + \frac{5}{100}$$

$$(iii) 803.207 = 8 \times 100 + 0 \times 10 + 3 \times 1 + 2 \times \frac{1}{10} + 0 \times \frac{1}{100} + 7 \times \frac{1}{1000}$$

$$= 800 + 3 + \frac{2}{10} + \frac{7}{1000}$$

$$(iv) 7.008 = 7 \times 1 + 0 \times \frac{1}{10} + 0 \times \frac{1}{100} + 8 \times \frac{1}{1000}$$

$$= 7 + \frac{8}{1000}$$

■ **Example 3.** Write each of the following as a decimal number:

(i) $30 + 6 + \frac{7}{10}$

(ii) $80 + 3 + \frac{7}{10} + \frac{5}{100}$

(iii) $500 + 9 + \frac{4}{10} + \frac{3}{1000}$

Solution.

$$(i) 30 + 6 + \frac{7}{10} = 3 \times 10 + 6 \times 1 + 7 \times \frac{1}{10}$$

$$= 36.7$$

$$(ii) 80 + 3 + \frac{7}{10} + \frac{5}{100} = 8 \times 10 + 3 \times 1 + 7 \times \frac{1}{10} + 5 \times \frac{1}{100}$$

$$= 83.75$$

$$(iii) 500 + 9 + \frac{4}{10} + \frac{3}{1000} = 5 \times 100 + 0 \times 10 + 9 \times 1 + 4 \times \frac{1}{10} + 0 \times \frac{1}{100} + 3 \times \frac{1}{1000}$$

$$= 509.403$$

■ **Example 4.** Write each of the following as decimals:

(i) Twenty nine and five tenths

(ii) Three hundred six and seven hundredths

(iii) Thirty seven point three zero six

(iv) Nine and twenty five thousandths

Solution.

$$\begin{aligned} \text{(i) Twenty nine and five tenths} &= 29 + \frac{5}{10} \\ &= 29 + 5 \times \frac{1}{10} = 29.5 \end{aligned}$$

$$\begin{aligned} \text{(ii) Three hundred six and seven hundredths} \\ &= 306 + \frac{7}{100} = 306 + 0 \times \frac{1}{10} + 7 \times \frac{1}{100} \\ &= 306.07 \end{aligned}$$

$$\text{(iii) Thirty seven point three zero six} = 37.306$$

$$\begin{aligned} \text{(iv) Nine and twenty five thousandths} \\ &= 9 + \frac{25}{1000} = 9 + 0 \times \frac{1}{10} + \frac{20}{1000} + \frac{5}{1000} \\ &= 9 + 0 \times \frac{1}{10} + 2 \times \frac{1}{100} + 5 \times \frac{1}{1000} \\ &= 9.025 \end{aligned}$$

■ **Example 5.** Write the numbers given in the following place value chart in decimal form:

	Hundreds (100)	Tens (10)	Ones (1)	Tenths ($\frac{1}{10}$)	Hundredths ($\frac{1}{100}$)	Thousandths ($\frac{1}{1000}$)
(i)	0	0	3	2	5	0
(ii)	0	3	0	0	2	5
(iii)	2	1	1	9	0	2

Solution.

$$\begin{aligned} \text{(i) Given number} &= 0 \times 100 + 0 \times 10 + 3 \times 1 + 2 \times \frac{1}{10} + 5 \times \frac{1}{100} + 0 \times \frac{1}{1000} \\ &= 3 + 2 \times \frac{1}{10} + 5 \times \frac{1}{100} = 3.25 \end{aligned}$$

$$\begin{aligned} \text{(ii) Given number} &= 0 \times 100 + 3 \times 10 + 0 \times 1 + 0 \times \frac{1}{10} + 2 \times \frac{1}{100} + 5 \times \frac{1}{1000} \\ &= 30 + 0 \times \frac{1}{10} + 2 \times \frac{1}{100} + 5 \times \frac{1}{1000} \\ &= 30.025 \end{aligned}$$

$$\begin{aligned} \text{(iii) Given number} &= 2 \times 100 + 1 \times 10 + 1 \times 1 + 9 \times \frac{1}{10} + 0 \times \frac{1}{100} + 2 \times \frac{1}{1000} \\ &= 211 + 9 \times \frac{1}{10} + 0 \times \frac{1}{100} + 2 \times \frac{1}{1000} \\ &= 211.902 \end{aligned}$$

■ **Example 6.** Between which two whole numbers on the number line do the given numbers lie? Which of these whole numbers is nearer to the number?



- (i) 0.7 (ii) 4.3 (iii) 7.9 (iv) 9.2

Solution.

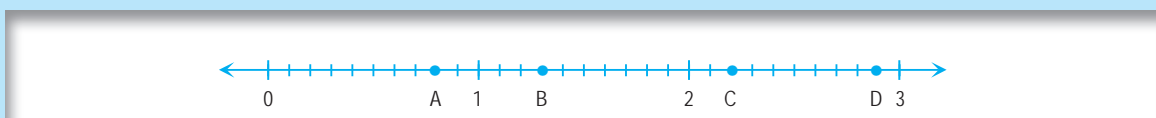
	Number	Lies between	Near to whole number
(i)	0.7	0 and 1	1
(ii)	4.3	4 and 5	4
(iii)	7.9	7 and 8	8
(iv)	9.2	9 and 10	9

Exercise 8.1

- Write each of the following decimal numbers in words:
 - 30.5
 - 0.03
 - 108.56
 - 47.20
 - 10.07
 - 0.032
 - 5.008
 - 26.039
- Write each of the following decimal numbers in the place value table:
 - 4.2
 - 0.3
 - 205.9
 - 0.29
 - 2.08
 - 4307.60
 - 7200.812
 - 38.007
- Write the following decimal numbers in the expanded form:
 - 123.7
 - 43.06
 - 509.306
 - 8.007
- Write each of the following as a decimal number:
 - $600 + 2 + \frac{8}{10}$
 - $200 + 60 + 5 + \frac{3}{10}$
 - $50 + \frac{1}{10} + \frac{6}{100}$
 - $700 + 20 + 5 + \frac{9}{100}$
 - $70 + 6 + \frac{7}{10} + \frac{9}{1000}$
 - $600 + 7 + \frac{3}{100} + \frac{6}{1000}$
- Write each of the following as decimals:
 - Two ones and five tenths
 - Two tens and nine tenths
 - Six hundred point eight
 - Two hundred five and five hundredths
 - Seven and fifteen thousandths
 - Nine hundred two and nine thousandths
- Write the number given in the following place value table in decimal form:

	Thousands (1000)	Hundreds (100)	Tens (10)	Ones (1)	Tenths $(\frac{1}{10})$	Hundredths $(\frac{1}{100})$	Thousandths $(\frac{1}{1000})$
(i)	7	1	0	2	3	0	6
(ii)		2	1	1	9	0	2
(iii)	3	0	5	3	0	1	5
(iv)			7	0	0	3	
(v)				5	4	0	
(vi)		7	1	9	0	2	8

- Show the following decimal numbers on the number line:
 - 0.4
 - 1.9
 - 1.1
 - 2.5
- Write the decimal numbers represented by the points A, B, C and D on the given number line:



- Between which two numbers in tenths place on the number line does each of the given number lie?
 - 0.06
 - 0.45
 - 0.18
 - 0.66
 - 0.92

FRACTIONS AND DECIMAL NUMBERS

Fractions with denominator 10, 100, 1000,... are called **decimal fractions**.

Decimal fractions can be written as decimal numbers and conversely decimal numbers can be written as decimal fractions.

Conversion of decimal fractions into decimal numbers

Count the number of zeros in the denominator and then count the same number of digits in the numerator starting from the unit digit and moving to the left, and then place the decimal point.

For example:

$$\frac{23}{10} = 2.3,$$

$$\frac{3}{10} = 0.3,$$

$$\frac{703}{100} = 7.03,$$

$$\frac{31}{100} = 0.31,$$

$$\frac{85301}{1000} = 85.301,$$

$$\frac{72}{1000} = 0.072$$

What about converse?

Conversion of decimal numbers to decimal fractions

In the denominator (i.e. below the division line) after 1, write the number of zeros equal to the number of decimal places in the given number.

For example:

$$32.7 = \frac{327}{10},$$

$$0.4 = \frac{4}{10},$$

$$3.54 = \frac{354}{100},$$

$$0.36 = \frac{36}{100},$$

$$85.473 = \frac{85473}{1000},$$

$$0.013 = \frac{13}{1000}$$

Remark

Any number of extra zeros written at the end of a decimal number does not change its value.

For example, 3.7 is same as 3.70 or 3.700

It is so because $3.7 = \frac{37}{10}$; $3.70 = \frac{370}{100} = \frac{37}{10}$; $3.700 = \frac{3700}{1000} = \frac{37}{10}$.

Note. Any whole number can be written as a decimal number by writing a decimal after its last (ones) digit and after it as many zeros as required.

For example:

37 is same as 37.0 or 37.00 etc.

Conversion of fractions into decimal numbers

First convert the given fraction to an equivalent decimal fraction, and then use the above method.

For example:

$$\frac{2}{5} = \frac{2 \times 2}{5 \times 2} = \frac{4}{10} = 0.4 ; \quad \frac{4}{25} = \frac{4 \times 4}{25 \times 4} = \frac{16}{100} = 0.16$$

Remarks

The decimal numbers which have a finite number of decimal places are called terminating decimal numbers. For example, 97.6, 3.74, 10.015 are all terminating decimals.