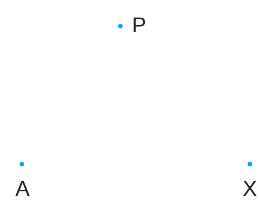
5 Basic Geometrical Concept

In this chapter, you will learn:

- Some basic concepts and terms of geometry—point, line, plane, line segment and ray
- Properties of points and lines in a plane
- Distinction between a line, a line segment and a ray
- Concept of angle
- Recognition of some plane figures—open and closed curves, polygons, triangles, quadrilaterals.
- Recognition of circle and its parts.

POINT

A small dot marked by a sharp pencil on a sheet of paper or a prick (hole) made by a fine needle on a paper are examples of a point. When we look the sky at night with naked eyes then the distant stars appear as points.

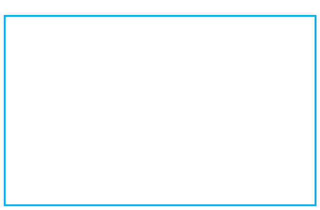


A point determines a location. It has neither length nor width nor thickness.

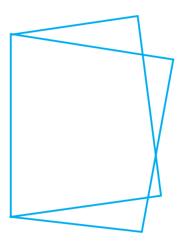
A point is denoted (or named) by a single capital letter of the alphabet such as P, A or X etc. and is read as 'point P', 'point A' or 'point X' etc.

LINE

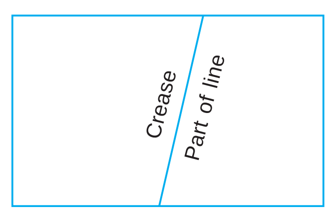
Fold a piece of paper and press the two parts together. On unfolding the two parts, we find that a *straight crease* is formed. This straight crease in the paper is an example of a portion of a line.



Piece of paper



Folding paper



Unfolding paper

A line has length only. It has no breadth or thickness.

The basic concept of a line is its straightness and it extends indefinitely in both directions.

By a line we mean the line in its totality and not just a part of it. Obviously, a line cannot be drawn wholly on a sheet of paper. In fact, only a portion of it is drawn and arrowheads are marked on its two ends

Line

to indicate that it extends in both directions. The two arrowheads in the opposite directions indicate that the ***length of a line is unlimited*** *i.e.* a line has no definite length.

A line has no end points.

A line can be denoted (or named) in two ways:

(*i*) By writing a single small letter of the alphabet such as *l* or *m* etc. by the side of it and is read as 'line *l*' or 'line *m*' etc.

(*i*) Line *l*

(*ii*) Line AB or $\overleftrightarrow{AB}$

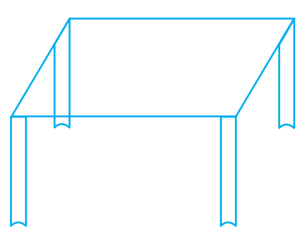
(*ii*) By taking two different points say A and B on the line, the line is denoted as $\overleftrightarrow{AB}$ or $\overleftrightarrow{BA}$ and is read as 'line AB'. The point A (or B) is said to lie on the line or that the line passes through the point A (or B).

Notice that between two points A and B on a line we can always find another point, say C, on the line AB. Moreover, this process can be continued indefinitely.

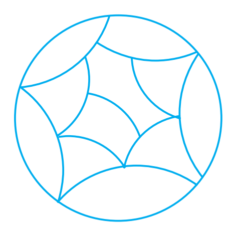
It follows that a line is made of an infinite (uncountable) number of points.

PLANE

A solid has a surface which may be *flat* or *curved*. For example, the surface of the top of a table is flat while the surface of a football is curved.



Table



Football

Flat surfaces are known as ***planes***.

A page of your notebook, surface of a wall, floor or ceiling of a room, surface of a blackboard or the top of a table are all examples of planes.

A plane has length and breadth. It has no thickness.

The basic concept of a plane is its flatness and it extends indefinitely in all directions.

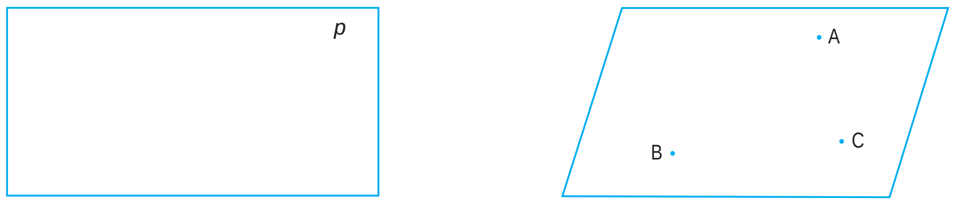
By a plane we mean the plane in its totality and not just a part of it. Obviously, a plane cannot be shown wholly. In practice, only a portion of a plane is shown in a figure. Usually, a plane is represented by a rectangle or a parallelogram.

A plane can be denoted (or named) in two ways:

(*i*) By writing a single small letter of the alphabet such as *p* or *q* etc. inside it and is read as 'plane *p*' or 'plane *q*' etc.

(*ii*) By taking three different points say A, B and C in the plane but not on the same line and is read as 'plane ABC'.

Since a plane extends indefinitely in all directions, so the **length and the breadth of a plane are unlimited** *i.e.* a plane has no definite length and no definite breadth.



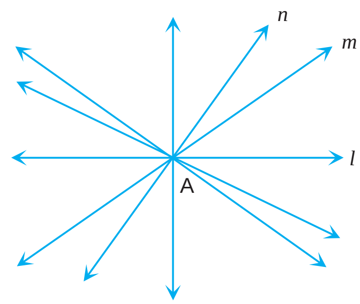
(*i*) Plane *p* (*ii*) Plane ABC

Properties of points and lines in a plane

Some of the properties of points and lines in a plane, which are universally true, are given below:

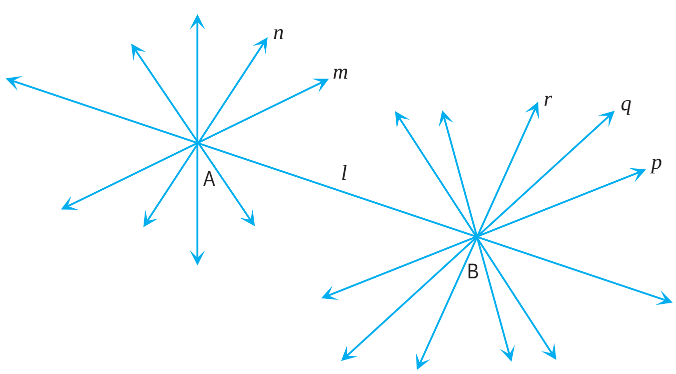
1. ***An unlimited number of lines can be drawn passing through a given point.***

Mark a point A on a sheet of paper. With a sharp pencil and (straight-edged) ruler, draw a line *l* passing through A. Draw another line *m* passing through A and continue this process. We find that an unlimited number of lines can be drawn in a plane passing through a given point on the plane.



2. ***There is exactly one line passing through two distinct given points in a plane.***

Mark two distinct points on a sheet of paper. With a sharp pencil and (straight-edged) ruler, draw lines *l*, *m*, *n*,... through A. Also draw lines *p*, *q*, *r*,... through B. Find the line (or lines) passing through both the points A and B.



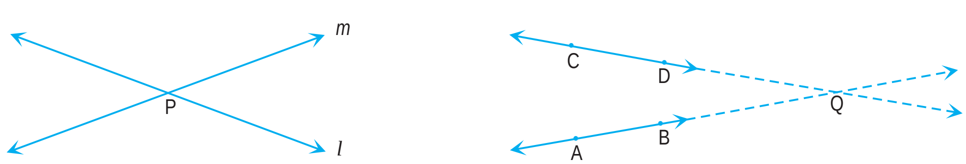
We observe that exactly one line (here line l) passes through both the given points A and B in a plane. Moreover, this line l lies wholly in the plane. Thus, two distinct points determine a unique line.

3. ***Two different lines in a plane either intersect at exactly one point or are parallel.***

Consider two different lines in a plane. There are two possibilities:

(*i*) the two lines intersect each other.

(*ii*) the two lines do not intersect each other.

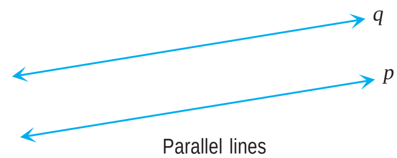


Intersecting lines

We observe that the lines l and m intersect each other at the point P. The point P which lies on both the lines is called the *point of intersection* of the lines l and m. Also observe that the lines AB and CD intersect each other and the point of intersection is Q.

We note that the lines p and q do not intersect each other and have no point in common. The lines p and q are called ***parallel***.

Thus, we find that two different lines in a plane either intersect at exactly one point or are parallel.



Parallel lines

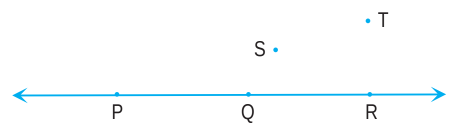
4. *Collinear points.*

Three or more points in a plane are called collinear points if and only if all of them lie on the same line.

(*i*) Collinear points

(*ii*) Non-collinear points

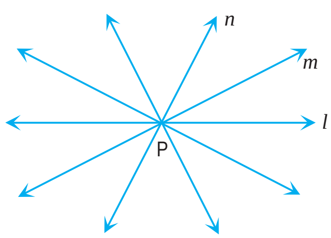
(*iii*) Collinear points



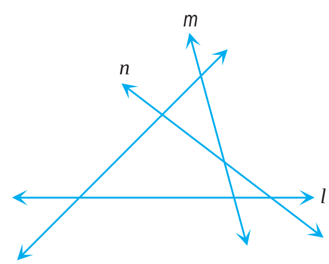
(*iv*) Non-collinear points

5. *Concurrent lines.*

Three or more lines (in a plane) are called ***concurrent lines*** *if and only if all of them pass through the same point, and this point is called the* ***point of concurrence****.*



Concurrent lines



Non-concurrent lines

LINE SEGMENT

Let us mark two distinct points A and B on a sheet of paper. There is exactly one line, say l, passing through the points A and B.

The portion of the line l between the points A and B (including the points A and B) is called the line segment AB.

It is denoted by $\overline{AB}$ or $\overline{BA}$ and is read as 'line segment AB'. The points A and B are called its *end points*.

Since there is exactly one line passing through the points A and B, therefore, there is exactly one line segment joining the points A and B. Hence, a line segment is completely known if its end points are known.

The length of the line segment $\overline{AB}$ is denoted by AB.

Length of line segment $\overline{AB}$ = the distance between the points A and B.

*Two or more line segments are called **equal** if and only if they have same length.*

Note that line segment AB and line segment BA are same.

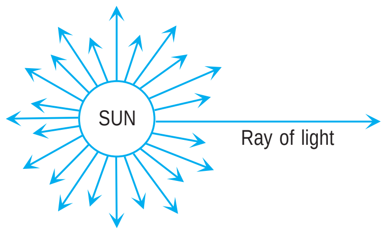
Thus, two line segments $\overline{AB}$ and $\overline{CD}$ are equal, written as $\overline{AB} = \overline{CD}$, if and only if AB = CD.

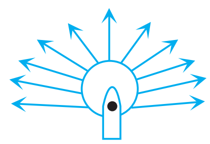
Point M is called ***mid-point*** of segment $\overline{AB}$ if and only if AM = MB.

*Two line segments are called **parallel/intersecting** if and only if the lines containing them are parallel/intersecting respectively.*

RAY

Sun emits rays of light. A lighted bulb gives rays of light. A **ray** starts from a fixed point and extends indefinitely in one direction.





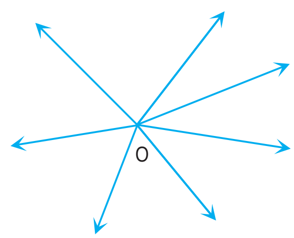
Mark a point O on a line l.

By a ray we mean the part of a line l that extends indefinitely in one direction from O including the point O, one ray starting from O is directed towards the right.

Ray

The point O is called **initial** (or **starting**) **point** of the ray. Thus, a ray has initial point as well as direction.

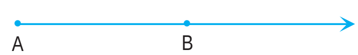
An unlimited number of rays can be drawn with a given initial point.



Rays with same initial point

There is exactly one ray which has O as its initial point and passes through a given point A. Thus, a ray is known completely if its initial point and one more point on it are known.

On this basis, we can name the ray as ray OA. It is denoted by $\overrightarrow{OA}$ and is read as 'ray OA'. *Ray OA means that the ray starts from O as its initial point and passes through the point A extending indefinitely in the direction from O to A.*



Ray AB

Ray BA

Recall, we know that there is no difference between line AB and line BA. Same is true about line segment AB and line segment BA. But we note that the two rays $\overrightarrow{AB}$ and $\overrightarrow{BA}$ are different. The ray AB has A as its initial point and its direction is from A to B while ray BA has B as its initial point and its direction is from B to A.

Opposite rays

Further, draw a line l and mark a point O on it, then mark two points A and B on this line on opposite sides of O. We get two rays $\overrightarrow{OA}$ and $\overrightarrow{OB}$ with same initial point O and extending indefinitely in opposite directions. Such rays are called **opposite rays**.

*Two rays are called **parallel/intersecting rays** if and only if the lines containing them are parallel/intersecting respectively.*

Distinction between a line, a line segment and a ray

Line	Line segment	Ray
A line has no end point	A line segment has two end points	A ray has one starting point
A line cannot be drawn wholly on a paper	A line segment can be drawn on a paper	A ray cannot be drawn wholly on a paper
A line has no definite length	A line segment has a definite length	A ray has no definite length
		
$\overleftrightarrow{AB}$ represents a line	$\overline{AB}$ is a line segment	$\overrightarrow{AB}$ represents a ray
$\overleftrightarrow{AB}$ and $\overleftrightarrow{BA}$ represent the same line	$\overline{AB}$ and $\overline{BA}$ are the same line segments	$\overrightarrow{AB}$ and $\overrightarrow{BA}$ represent two different rays

Remark

Till now we were using the symbols $\overleftrightarrow{AB}$, $\overline{AB}$, AB and $\overrightarrow{AB}$ for line AB, line segment AB, length of line segment AB and ray AB respectively. Usually, we use the symbol AB for all the four concepts (unless stated otherwise) for convenience. It will be clear from the context which of above four concepts we are referring to.

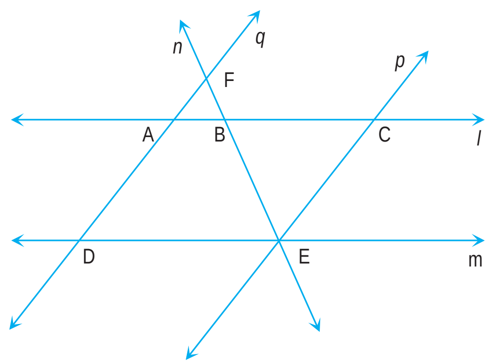
Example 1. From the given figure, write

(*i*) all pairs of parallel lines

(*ii*) all pairs of intersecting lines

(*iii*) concurrent lines and point of concurrence

(*iv*) collinear points.



Solution.

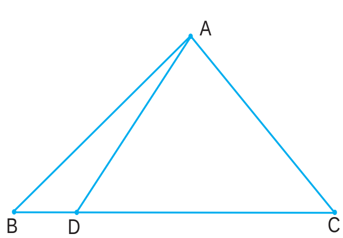
(*i*) l, m; p, q

(*ii*) l, p; l, q; l, n; m, p; m, n; m, q; n, p; n, q

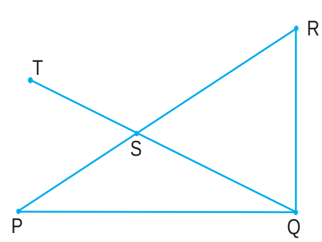
(*iii*) m, p, n; point of concurrence is E

(*iv*) A, B, C; D, A, F; E, B, F.

Example 2. Count the number of line segments drawn in each of the following figures and name them:



(*i*)



(*ii*)

Solution.

(*i*) Line segments AB, BD, AD, DC, BC and AC; six.

(*ii*) Line segments PQ, QR, PS, SR, PR, QS, ST and QT; eight.

ACTIVITY 3

1. Take four distinct collinear points A, B, C and D in plane (page of your copy)

(*i*) Find the number of lines that can be drawn passing through these points.

(*ii*) Name the line segments.

2. Take four distinct points A, B, C and D, out of which three points are collinear.

(*i*) Find the number of lines that can be drawn passing through these points.

(*ii*) Name the line segments.

3. Take four distinct points A, B, C and D, no three of which are collinear.

(*i*) Find the number of lines that can be drawn passing through these points.

(*ii*) Name the line segments.

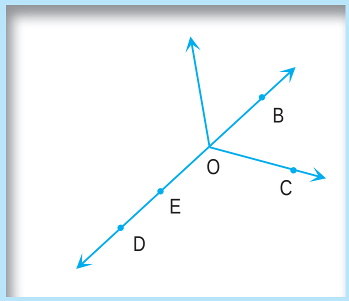
Exercise 5.1

1. How many lines can be drawn through a given point?

2. How many lines can be drawn through two distinct given points?

3. How many lines can be drawn through three collinear points?

4. Mark three non-collinear points A, B and C in your notebook. Draw lines through these points taking two at a time and name these lines. How many such different lines can be drawn?



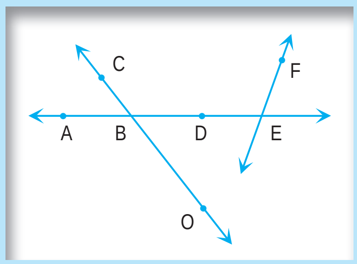
5. Use the adjoining figure to name:

(*i*) Five points.

(*ii*) A line.

(*iii*) Four rays.

(*iv*) Five line segments.



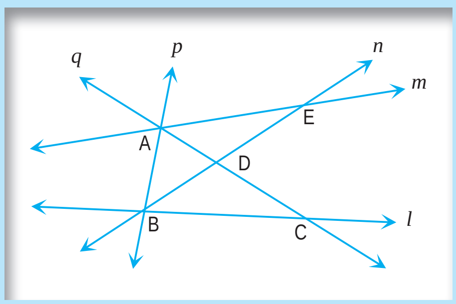
6. Use the adjoining figure to name:

(*i*) Line containing point E.

(*ii*) Line passing through A.

(*iii*) Line on which point O lies.

(*iv*) Two pairs of intersecting lines.

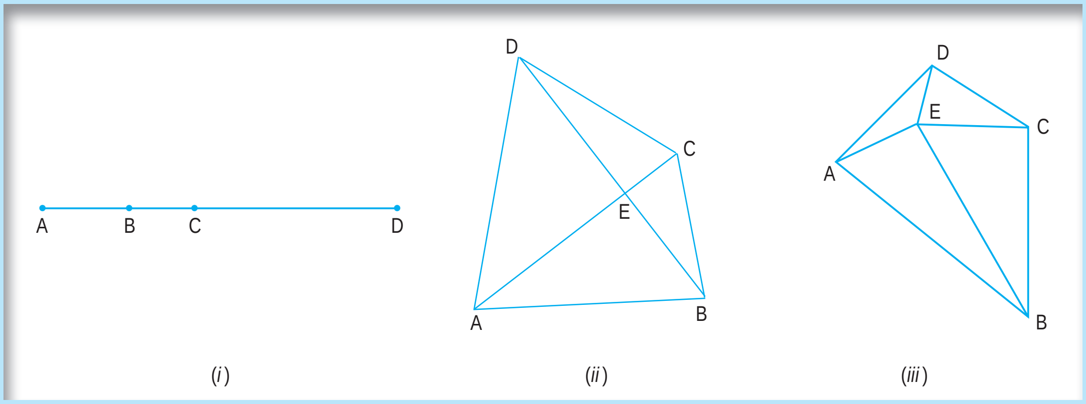


7. From the adjoining figure, write

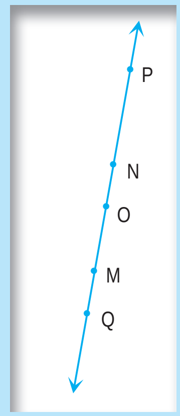
(*i*) collinear points.

(*ii*) concurrent lines and their points of concurrence.

8. Count the number of segments drawn in each of the following figures and name them:



9. Consider the adjoining figure of the line $\overleftrightarrow{MN}$. State whether the following statements are true (T) or false (F) in context of the given figure.



(*i*) Q, M, O, N and P are points on the line $\overleftrightarrow{MN}$.

(*ii*) M, O and N are points on the line segment $\overline{MN}$.

(*iii*) M and N are end points of the segment $\overline{MN}$.

(*iv*) O and N are end points of the segment $\overline{OP}$.

(*v*) M is a point on the ray $\overrightarrow{OP}$.

(*vi*) M is one of the end point of the segment $\overline{QO}$.

(*vii*) Ray $\overrightarrow{OP}$ is same as ray $\overrightarrow{OM}$.

(*viii*) Ray $\overrightarrow{OM}$ is not opposite to ray $\overrightarrow{OP}$.

(*ix*) Ray $\overrightarrow{OP}$ is different from ray $\overrightarrow{QP}$

(*x*) O is not an initial point of ray $\overrightarrow{OP}$.

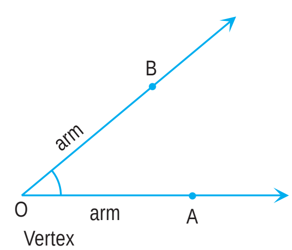
(*xi*) N is the initial point of $\overrightarrow{NP}$ and $\overrightarrow{NM}$.

ANGLE

The adjoining diagram shows an angle.

*An **angle** is a figure formed by two rays having same initial point.*

The initial point is called its **vertex** and the two rays are called its **arms** or **sides** or **legs**.



In the adjoining figure, O is the vertex and the rays OA and OB are the arms of the angle. Note that each arm has a particular direction. Usually, the arms are joined by a small circular arc near the vertex as shown in the figure.

We use the symbol '∠' to denote an angle. The given angle is denoted by ∠AOB or ∠BOA and is read as 'angle AOB' or 'angle BOA'. ***Notice that in either case the vertex of the angle is written in the middle***.

Sometimes, a given angle is denoted or named simply as ∠P or angle P (see the adjoining figure).