12 | Algebra

INTRODUCTION

So far, we have studied two branches of mathematics—

Arithmetic and Geometry. **Arithmetic** deals with numbers, operations on numbers, properties of numbers and the application of this knowledge in solving some real life problems. **Geometry** deals with figures/shapes and their properties.

In this chapter, we shall study another branch of mathematics named **algebra**. In algebra, we use letters for numbers. By using letters, we can talk about any number and not just a particular number. Use of letters will allow us to write rules and formulas in a general way. Since letters stand for numbers, arithmetic operations can be performed on them as on numbers. Further, letters may stand for unknown quantities. By learning methods of determining unknown, we shall develop powerful tools for solving many problems from daily life.

Variables

The word 'variable' means something that can vary *i.e.* change. The value of a variable is not fixed. It can take diffenent values. We shall learn the idea of variables through matchstick patterns and word problems.

Matchstick Patterns

Patterns can be made by using matchsticks to form letters of English alphabet such as L, T, V, C, U, F, E, etc. or geometrical shapes such as equilateral triangle, square etc.

1. Look at the following pattern of the letter L formed by matchsticks:

(i)   (ii)   (iii)
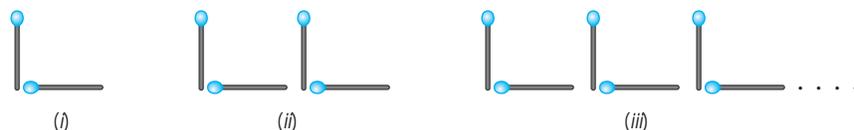
In fig. (*i*), using 2 matchsticks one L is formed.

In fig. (*ii*), using 4 matchsticks two L's are formed.

In fig. (*iii*), using 6 matchsticks three L's are formed and so on. We prepare table 1 as under:

Table 1

Number of L's formed	1	2	3	4	5	6	7	.....
Number of matchsticks required	2	4	6	8	10	12	14	.....

We observe that the number of matchsticks required is double the number of L's formed.

Number of matchsticks required = 2 × number of L's formed.

If we write the letter n for the number of L's formed, then number of matchsticks required = 2 × n.

Instead of writing 2 × n, we usually write $2n$. Note that $2n$ is same as 2 × n.

For $n = 1$, number of matchsticks = 2 × 1 = 2,

For $n = 2$, number of matchsticks = 2 × 2 = 4,

For $n = 3$, number of matchsticks = 2 × 3 = 6 and so on.

Thus, the number of matchsticks required = 2n.

*Here, letter n is an example of a **variable** (**literal** or **generalized number**). Its value is not fixed, it can take any value 1, 2, 3, 4, ... Remember, a variable is a number which does not have a fixed value.*

2. Look at the following pattern of letter C formed by matchsticks.

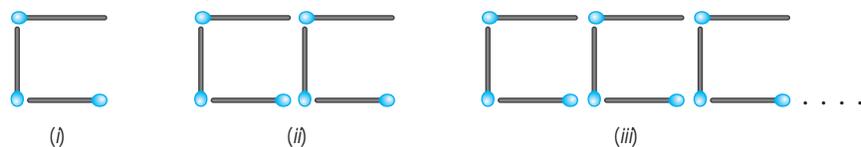To make one C, three matchsticks are used.

(i)   (ii)   (iii)

Table 2 gives the number of matchsticks required to make a pattern of C's.

Table 2

Number of C's formed	1	2	3	4	5	6	...
Number of matchsticks required	3	6	9	12	15	18	...

If we write the letter m for the number of C's formed, then the number of matchsticks required = $3m$.

Here, the letter m is an example of a variable. It can take any value 1, 2, 3, 4, ...

3. Look at the following pattern of letter E formed by matchsticks.
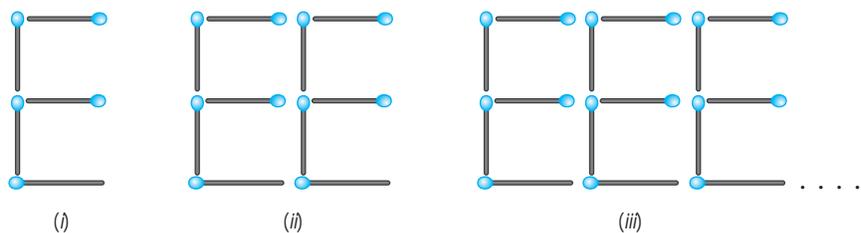
To make one E, five matchsticks are used.

(i)   (ii)   (iii)

Table 3 gives the number of matchsticks required to make a pattern of E's.

Table 3

Number of E's formed	1	2	3	4	5	6	.....
Number of matchsticks required	5	10	15	20	25	30	.....

If we write the letter p for the number of E's formed, then the number of matchsticks required = $5p$.

Here, the letter p is an example of a variable. It can take any value 1, 2, 3, 4, ...

4. Look at the following pattern of squares formed by matchsticks. The squares are not separate. Two neighbouring squares have a common matchstick.
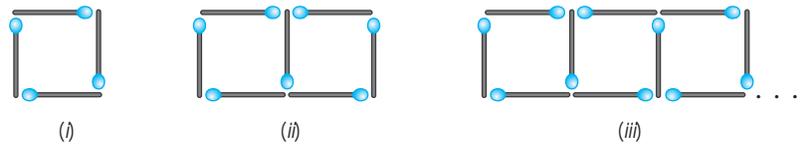

(*i*)   (*ii*)   (*iii*)

Table 4 gives the number of matchsticks required to make the given pattern of squares.

Table 4

Number of squares formed	1	2	3	4	5	.....
Number of matchsticks required	4	7	10	13	16	.....

Note that $4 = 3 \times 1 + 1$, $7 = 3 \times 2 + 1$, $10 = 3 \times 3 + 1$, ...

If we write the letter x for the number of squares formed, then the number of matchsticks required
$$= 3x + 1.$$

Here, the letter x is an example of a variable. It can take any value 1, 2, 3, 4, ...

Some examples of variables through word problems

■ **Example 1.** Students want to buy notebooks from school bookstore. If the price of one notebook is ₹ 14, how much money should a student carry when he/she goes to the bookstore to buy notebooks?

Solution. The money a student should carry will depend on how many notebooks the student wants to buy.

If the letter m stands for the number of notebooks a student wants to buy, then total cost (in ₹) of m notebooks is given by the rule:

The total cost (in ₹) = $14m$.

Here, m is a variable and it can take any value 1, 2, 3, 4, ...

In particular, if $m = 5$, then the cost = ₹ $(14 \times 5) = $ ₹ 70.

■ **Example 2.** For the Republic Day celebrations in a school, students are going to perform a mass drill in the presence of the chief guest. They stand 8 in a row. How many students are there in the drill?

Solution. The number of students in the drill will depend upon the number of rows. If there are r rows, then total number of students in the drill = $8r$.

Here, letter r is a variable and it can take any value 1, 2, 3, 4, ...

In particular, if $r = 10$, then the number of students in the drill = $8 \times 10 = 80$.

■ **Example 3.** Sarita has 10 more marbles than Ameena has in her collection. Can you write Sarita's marbles in terms of Ameena's marbles?

Solution. Sarita's marbles depend upon Ameena's marbles. If Ameena has x marbles, then number of marbles that Sarita has = $x + 10$.

Here, the letter x is a variable and it can take any value 1, 2, 3, ...

In particular, if Ameena has 20 marbles, then Sarita has $20 + 10$ *i.e.* 30 marbles.

■ **Example 4.** Raju and Balu are brothers. Balu is younger than Raju by 3 years. Can you write Balu's age in terms of Raju's age?

Solution. Balu's age depends upon Raju's age. If Raju's age is x years, then

Balu's age = $(x - 3)$ years.

Here, the letter x is a variable and it can take any value greater than 3 *i.e.* $x = 4, 5, 6, ...$

In particular, when Raju is 12 years old then Balu's $(12 - 3)$ years *i.e.* 9 years old.

■ **Example 5.** A parrot flies 400 m in one minute. Can you write the distance in km covered by the parrot in terms of its flying time in minutes?

Solution. Distance covered by the parrot in one minute = 400 m

$$= \frac{400}{1000} \text{ km} = \frac{2}{5} \text{ km}.$$

The total distance covered by the parrot depends upon its flying time. If the parrot flies for t minutes, then total distance covered by the parrot $= \frac{2}{5}t$ km.

Here, the letter t is a variable.

Constants and variables

In algebra, we use two types of symbols — constants and variables.

Constant

*A symbol which has a fixed value is called a **constant**.*

Thus, each of $7, -3, 0, \frac{2}{3}, -\frac{5}{7}, 2\frac{3}{8}$ etc. is a constant.

Variable

*A symbol (or letter) which can be given various numerical values is called a **variable**.*

Thus, a variable is a number which does not have a fixed value. Variables are generalised numbers or unknown numbers.

As variables are usually denoted by letters, so variables are also known as **literal** (or **literal numbers**).

> ### Remark
>
> In $4x$, 4 is constant and x is variable but together $4x$ is variable.
>
> When $x = 2$, $4x = 4 \times 2 = 8$.
>
> When $x = 3$, $4x = 4 \times 3 = 12$ etc.
>
> Thus, when x takes various values, $4x$ also takes corresponding various values.
>
> Similarly, $x + 4$, $x - 4$, $4 - x$, $\frac{x}{4}$ and $\frac{4}{x}$ are variables.
>
> *Hence, a combination of a constant and a variable is a variable.*

Exercise 12.1

1. Find the rule which gives the number of matchsticks required to make the following matchstick patterns. Use a variable (literal) to write the rule.

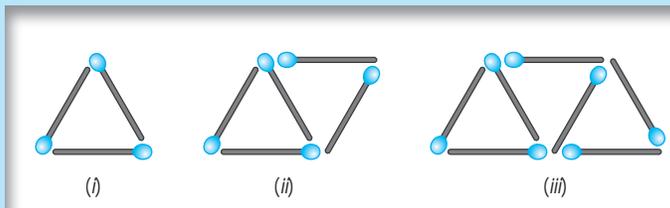   (*i*) A pattern of letter T as ⊤

   (*ii*) A pattern of letter V as ∨

   (*iii*) A pattern of letter Z as Ƶ

   (*iv*) A pattern of letter U as ⊔

   (*v*) A pattern of letter F as F

   (*vi*) A pattern of letter S as S

2. Cadets are marching in a parade. There are 6 cadets in a row. What is the rule which gives the number of cadets, given the number of rows? (Use n for the number of rows.)

3. If there are 24 mangoes in a box, how will you write the number of mangoes in terms of the number of boxes? (Use b for the number of boxes.)

4. The teacher distributes 5 toffees to each student. Can you tell how many toffees are needed, given the number of students? (Use s for the number of students.)

5. Anuradha is drawing a dot Rangoli (a beautiful pattern of lines joining dots). She has 8 dots in a row. How many dots will her Rangoli have for r rows? How many dots are there if there are 12 rows?

6. Anu and Meenu are sisters. Anu is 5 years younger than Meenu. Can you write Anu's age in terms of Meenu's age? Take Meenu's age as x years.

7. Oranges are to be transferred from larger boxes to smaller boxes. When a larger box is emptied, the oranges from it fill 3 smaller boxes and still 7 oranges are left. If the number of oranges in a small box are taken to be x, then what is the number of oranges in the larger box?

8. Harsha's score in Mathematics is 15 more than three-fourth of her score in Science. If she scores x marks in Science, find her score in Mathematics?

9. Look at the following matchstick pattern of equilateral triangles. The triangles are not separate. Two neighbouring triangles have a common matchstick. Observe the pattern and find the rule that gives the number of matchsticks.

(i)   (ii)   (iii)

10. Look at the following matchstick pattern of letter A. The A's are not separate. Two neighbouring A's have two common matchsticks. Observe the pattern and find the rule that gives the number of matchsticks.

(i)   (ii)   (iii)
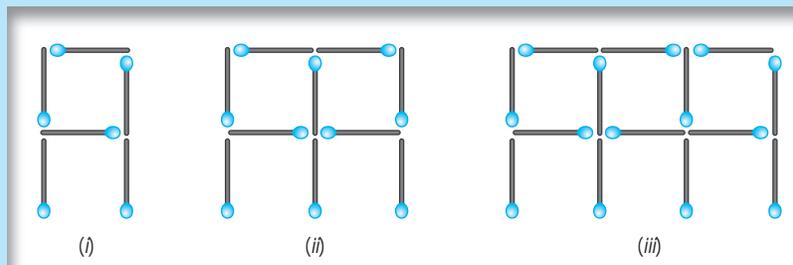
USES OF VARIABLES

We have already learnt certain rules in geometry and arithmetic. Here, we shall see that these rules become easier to express by using variables.

Rules from geometry

1. Perimeter of a square

The perimeter of a square = sum of the lengths of the sides of the square

$\qquad$ = 4 times the length of the side of the square

$\qquad$ = $4 \times a = 4a$, where a is length of a side.