● —— SUPPLEMENT TO CHAPTER 8 —— ●

DIVISION BY VEDIC METHODS

In Vedic Mathematics, the various methods used for division are :

 (*i*) *Vilokanam sūtra*.

 (*ii*) *Nikhilam sūtra*—All from 9 and the last from 10.

 (*iii*) *Parāvartya sūtra*—Transpose and Adjust.

 (*iv*) *Dhvajānka sūtra*—Vertically and crosswise and on top of the flag.

Out of these methods, the most general method of division is 'vertically and crosswise and on top of the flag'. This method is also called straight division. By this method, we can divide numbers of any size by numbers of any size. *Sankaracārya Bhārati* Krsna Tirthaji called this "the crowning gem of Vedic Mathematics" we shall try to learn the general method of division (*Dhvajānka sūtra*). Before beginning with the method there are four names connected with division which must be learnt and understood. These are, divisor, dividend, quotient and remainder. The following rhyme may help :

The divisor is the number that divides the dividend, the answer's in the quotient, the remainder's at the end.

For example : 19 ÷ 6 = 3, remainder 1

(19 = 6 × 3 + 1), 6 is the divisor, 19 is the dividend, 3 is the quotient and 1 is the remainder.

DIVISION BY ONE DIGIT NUMBERS

In the division of any number by a one digit number, we use the normal way of division. In Vedic Mathematics, the way of writing this division is different. The following examples show this method :

Example 1. *Divide 773 by 4 i.e.* 4 $\lfloor$ *773* .

Solution.

$$
\begin{array}{c|cccc}
4 & 7 & 7 & 3 \\
  &   & 3 & 1 \\
\hline
  & 1 & 9 & 3 \vdots 1
\end{array}
$$

7 ÷ 4 = 1, remainder 3   (∵ 7 = 4 × 1 + 3); Write 1 in quotient (below division line) and annex remainder 3 to the next digit 7 in the dividend to make 37.

37 ÷ 4 = 9, remainder 1; write 9 in quotient (as next digit), annex remainder 1 to next digit 3, to make 13.

∴ Quotient is 193 and remainder is 1.

13 ÷ 4 = 3, remainder 1. Write 3 in the quotient.

Example 2. *Divide 7436 by 5 i.e.* 5 $\lfloor$ *7436* .

Solution.

$$
\begin{array}{c|cccc}
5 & 7 & 4 & 3 & 6 \\
  &   & 2 & 4 & 3 \\
\hline
  & 1 & 4 & 8 & 7 \vdots 1
\end{array}
$$

7 ÷ 5 = 1, remainder 2.

Annex remainder 2 to next digit 4, to make 24.

24 ÷ 5 = 4, remainder 4.

Annex remainder 4 to next digit 3, to make 43.

43 ÷ 5 = 8, remainder 3.

Annex remainder 3 to next digit 6, to make 36

36 ÷ 5 = 7, remainder 1.

∴ Quotient is 1487 and remainder 1.

Example 3. *Find* 8⌐5032 .

Solution.

8	50	3	2
	2	7	
	6	2	9 ┊ 0

$50 \div 8 = 6$, remainder 2.
Annex remainder 2 to next digit 3, to make 23.
$23 \div 8 = 2$, remainder 7.
Annex remainder 7 to next digit 2, to make 72.
$72 \div 8 = 9$, remainder $= 0$.

∴ Quotient = 629, remainder = 0.

Example 4. *Find* 9⌐73544 .

Solution.

9	73	5	4	4
	1	6	1	
	8	1	7	1 ┊ 5

∴ Quotient = 8171 and remainder = 5.

$73 \div 9 = 8$, remainder 1.
Annex remainder 1 to 5, to make 15
$15 \div 9 = 1$, remainder 6.
Annex remainder 6 to 4, to make 64.
$64 \div 9 = 7$, remainder $= 1$.
Annex remainder 1 to 4, to make 14.
$14 \div 9 = 1$, remainder 5.

Exercise 8.1 (S)

Find the quotient and the remainder in the following :

1. 3⌐786

2. 4⌐9561

3. 5⌐7362

4. 6⌐2330

5. 7⌐8703

6. 7⌐5049

7. 8⌐7321

8. 9⌐2330

9. 9⌐70642

10. 9⌐77077

11. 8⌐3030

12. 9⌐4261

STRAIGHT DIVISION (*DHVAJĀNKA SŪTRA*)

By *Dhvajānka Sūtra* (Vertically and crosswise and on top of the flag), we can perform the division a number of any size by a number of any size. We shall try to learn it through some examples.

Example 1. *Divide 7332 by 64 i.e.* 64⌐7332 .

Solution. From the divisor 64, we write only the first digit *i.e.* 6 in division column and put the other digit *i.e.* 4 on top of the flag as shown alongside. As one digit is put on top of the flag, we allot one place at the right end of the dividend to the remainder portion of the answer and make it off from the other digits by a vertical (dotted) line.

	4				
6	7	3	3 ┊	2	
		1	3 ┊ 5		
	1	1	4 ┊	36	

The entire division is to be done by 6.

The procedure is explained below:

Divide 7 by 6 and get 1 as quotient and 1 as remainder. Write 1 as the first quotient digit below the division line and prefix the remainder 1 up just before 3 (the next digit of dividend), to make gross dividend 13. From this, subtract the product of the flag digit 4 and the first digit of the quotient 1 *i.e.* 4 × 1. Thus, 13 − 4 × 1 = 9, which is actual dividend. Now 9 ÷ 6 = 1, remainder 3. Put quotient digit 1 and remainder 3 in the respective places as before. The next gross dividend is 33. From this, subtract product

of flag digit and the second quotient digit *i.e.* 4 × 1. Thus, 33 – 4 × 1 = 29, which is the next actual dividend.

29 ÷ 6 = 4, remainder 5. Put quotient digit 4 and remainder 5 in their respective places. The division is finished but the gross remainder is 52. From this, subtract the product of the flag digit and the next digit of quotient *i.e.* 4 × 4. Thus, 52 – 4 × 4 = 36, which is the actual remainder.

Hence, quotient is 114 and remainder is 36.

Example 2. *Find* 75⌐3276 .

Solution. By *Dhvajānka Sūtra*, we have :

The procedure is as follows :

From the divisor 75, write 7 in the division column and put 5 on the top of the flag. We allot one place at the right end of the dividend to the remainder portion of the answer. Cut off the last digit 6 of the dividend from the other digits by a vertical line for the remainder portion.

$$
\begin{array}{c|ccc|c}
5 & & & & \\
\hline
7 & 32 & 7 & \vdots & 6 \\
& & 4 & \vdots\, 6 & \\
\hline
& 4 & 3 & \vdots & 51 \\
\end{array}
$$

32 ÷ 7 = 4, remainder 4. Write first quotient digit 4 and remainder digit 4 at their respective places (explained in example 1).

Gross dividend = 47.

From 47, subtract the product of flag digit 5 and the quotient digit 4 *i.e.* 5 × 4. Thus 47 – 5 × 4 = 27.

Next actual dividend is 27.

27 ÷ 7 = 3, remainder 6.

Write next digit 3 of quotient and remainder 6 at the respective places. Division is finished.

Gross remainder = 66.

From 66, subtract the product of flag digit 5 and the second quotient digit 3 *i.e.* 5 × 3. Thus, 66 – 5 × 3 = 51.

∴   Actual remainder = 51.

Hence, quotient is 43 and remainder is 51.

Example 3. *Divide 38985 by 73.*

Solution. By *Dhvajānka Sūtra*, we have :

Procedure :

Put 7 in the divisor column and 3 on top of the flag. Cut off the last digit 5 of the dividend by a vertical line.

$$
\begin{array}{c|cccc|c}
3 & & & & & \\
\hline
7 & 38 & 9 & 8 & \vdots & 5 \\
& & 3 & 3 & \vdots\, 1 & \\
\hline
& 5 & 3 & 4 & \vdots & 3 \\
\end{array}
$$

38 ÷ 7 = 5, remainder 3

Gross dividend is 39.

Actual dividend = 39 – 3 × 5 = 24.

24 ÷ 7 = 3, remainder 3.

Next gross dividend = 38.

Next actual dividend = 38 – 3 × 3 = 29.

29 ÷ 7 = 4, remainder 1.

Gross remainder = 15.

Actual remainder = 15 – 3 × 4 = 3.

Hence, quotient = 534 and remainder = 3.

Example 4. *Divide 90287 by 53.*

Solution. By *Dhvajānka Sūtra*, we have :

    Procedure :

9 ÷ 5 = 1, remainder 4.

Gross dividend = 40.

Actual dividend = 40 – 3 × 1 = 37.

37 ÷ 5 = 7, remainder 2.

Next gross dividend = 22.

Next actual dividend = 22 – 3 × 7 = 1.

1 ÷ 5 = 0, remainder 1.

Next gross dividend = 18.

Next actual dividend = 18 – 3 × 0 = 18.

18 ÷ 5 = 3, remainder = 3.

Gross remainder = 37.

Actual remainder = 37 – 3 × 3 = 28.

Hence, quotient = 1703 and remainder = 28.

	3					
5	9	0	2	8	⋮	7
		4	2	1	⋮ 3	
	1	7	0	3	⋮	28

Caution : The actual dividend cannot be negative at any stage of division. Also the actual remainder cannot be negative.

Exercise 8.2 (S)

Find the quotient and the remainder in the following divisions :

1. 41$\overline{|891|}$ **2.** 51$\overline{|846|}$ **3.** 24$\overline{|5353|}$ **4.** 81$\overline{|8819|}$

5. 71$\overline{|4354|}$ **6.** 56$\overline{|9534|}$ **7.** 56$\overline{|95347|}$ **8.** 92$\overline{|18745|}$

ALTERED REMAINDERS

The question arises! How many three's are there in 13? Since 13 = 3 × 4 + 1, the answer is 4 and remainder 1. But this is not the only answer. The various possibilities are :

    13 = 3 × 4 + 1,

    13 = 3 × 3 + 4,

    13 = 3 × 2 + 7,

    13 = 3 × 1 + 10 and

    13 = 3 × 0 + 13.

Thus, we get five different remainders 1, 4, 7, 10 and 13. Usually, we give the conventional answer 1 which is useful for most purposes but there are occasions where an altered remainder is required for Straight Division.

Example 1. *Give the answer together with next two altered remainders for 26 ÷ 4.*

Solution. 26 = 4 × 6 + 2,

        26 = 4 × 5 + 6,

        26 = 4 × 4 + 10.

Thus, on dividing 26 by 4, the remainder is 2 and two altered remainders are 6 and 10.

Thus, 26 ÷ 4 = 6, remainder 2; 5 remainder 6; 4 remainder 10.

These can be written as $6/2, 5/6, 4/10$; remainders are written after slant stroke.

Example 2. *Give answer together with next three altered remainders for the following divisions:*

    (i) $19 \div 3$ (ii) $36 \div 6$

Solution. (i) $19 = 3 \times 6 = 1,$

        $19 = 3 \times 5 + 4,$

        $19 = 3 \times 4 + 7$ and

        $19 = 3 \times 3 + 10.$

    ∴   The answer together with next three altered remainders are :

        $6/1, 5/4, 4/7, 3/10$

    (ii) $36 = 6 \times 6 + 0,$

           $36 = 6 \times 5 + 6,$

           $36 = 6 \times 4 + 12$ and

           $36 = 6 \times 3 + 18.$

  ∴ The answer together with next three altered remainders are :

        $6/0, 5/6, 4/12, 3/18.$

Exercise 8.3 (S)

1. Give the answer together with next two altered remainders for the following divisions :

    (i) $31 \div 3$ (ii) $62 \div 7$ (iii) $47 \div 5$

2. Give the answer together with next three altered remainders for the following divisions :

    (i) $67 \div 9$ (ii) $48 \div 8$ (iii) $39 \div 6$

STRAIGHT DIVISION WITH ALTERED REMAINDERS

In Straight Division, the key to the problem is look ahead at each stage to see whether or not the remainder needs to be altered. At each stage, before the quotient digit is written with its remainder, the next step of subtraction must be checked. If the subtraction leads to a minus number, then an altered remainder is required at the previous step. The reader is advised to have sufficient practice involving altered remainders. To learn the Straight Division with altered remainders, study the following examples :

Example 1. *Divide 4230 by 37.*

Solution. By *Dhvajānka sūtra*, we have :

    Procedure :

        $4 \div 3 = 1$, remainder 1

    Write quotient digit 1 and remainder 1 at their respective places.

$$
\begin{array}{c|cccc}
7 & & & & \\
\hline
3 & 4 & 2 & 3 & 0 \\
& 1 & 2 & 4 & \\
\hline
& 1 & 1 & 4 & 12 \\
\end{array}
$$

Gross dividend = 12,

actual dividend = 12 – 7 × 1 = 5

5 ÷ 3 = 1, remainder 2.

Write quotient digit 1 and remainder 2 at their respective places.

Next gross dividend = 23,

next actual dividend = 23 – 7 × 1 = 16

16 ÷ 3 = 5, remainder is 1.

Gross remainder = 10,

actual remainder = 10 – 7 × 5 = – 25, not allowed.

Therefore, we need altered remainder at the previous step.

16 = 3 × 4 + 4, so quotient is 4 and remainder is 4.

Write quotient digit 4 and remainder 4 at their respective places.

Gross remainder = 40,

actual remainder = 40 – 7 × 4 = 12.

Hence, 4230 ÷ 37 = 114, remainder 12

   i.e.   quotient = 114 and remainder = 12.

Example 2. *Divide 7816 by 22.*

Solution. By *Dhvajānka sūtra*, we have :

   Procedure :

       7 ÷ 2 = 3, remainder 1

Write quotient digit 3 and remainder 1 at their respective places.

Gross dividend = 18,

actual dividend = 18 – 2 × 3 = 12, which is +ve.

       12 ÷ 2 = 6, remainder 0.

Next gross dividend = 01 = 1,

next actual dividend = 1 – 2 × 6 = –11, which is –ve, so not allowed.

Therefore, we need an alternate remainder.

       12 ÷ 2 = 5, remainder 2 *i.e.* 12 = 2 × 5 + 2.

Write quotient digit 5 and remainder 2 at their respective places.

Thus, next gross dividend = 21,

next actual dividend = 21 – 2 × 5 = 11.

       11 ÷ 2 = 5, remainder 1.

Write quotient digit 5 and remainder 1 at their respective places.

Gross remainder = 16,

actual remainder = 16 – 2 × 5 = 6.

Hence, 7816 ÷ 22 gives quotient = 355 and remainder = 6.

2				
2	7	8	1 ¦	6
	1	2	¦ 1	
	3	5	5 ¦	6

Example 3. *Divide 435 by 23.*

Solution. By *Dhvajānka sūtra*, we have :

   Procedure :

       4 ÷ 2 = 2, remainder 0.

Gross dividend = 03 = 3.

3			
2	4	3 ¦	5
	2	¦ 4	
	1	8 ¦	21

Actual dividend = 3 – 3 × 2 = –6, but negative dividend is not allowed, therefore, we need an altered remainder at the previous step.

Now 4 = 2 × 1 + 2, so quotient is 1 and remainder is 2.

Write these at their respective places.

Gross dividend = 23,

actual dividend = 23 – 1 × 3 = 20

20 ÷ 2 = 10, remainder 0 but quotient cannot be 2-digit number, therefore, we need altered remainder.

20 = 2 × 9 + 2, quotient is 9 and remainder 2.

Gross remainder = 25,

actual remainder = 25 – 3 × 9 = –2 but remainder cannot be negative, therefore, we still need an alternate remainder.

20 = 2 × 8 + 4, quotient is 8 and remainder is 4.

Write these at their respective places.

Gross remainder = 45,

actual remainder = 45 – 3 × 8 = 21.

Hence, quotient = 18 and remainder = 23.

Example 4. *Divide 6290 by 38.*

Solution. By *Dhvajānka sūtra*, we have :

Procedure :

6 ÷ 3 = 2, remainder 0.

Gross dividend = 02 = 2,

actual dividend = 2 – 8 × 2 = –14, not allowed.

Therefore, we need altered remainder at the previous step.

6 = 3 × 1 + 3, quotient is 1 and remainder is 3.

Write quotient digit 1 and remainder 3 as their respective places.

Gross dividend = 32,

actual dividend = 32 – 1 × 8 = 24.

24 ÷ 3 = 8, remainder = 0.

Next gross dividend = 09 = 9,

next actual dividend = 9 – 8 × 8 = –55, not allowed.

Therefore, we need altered remainder at previous step.

24 = 3 × 7 + 3, quotient is 7 and remainder is 3.

So gross dividend = 39,

actual dividend = 39 – 8 × 7 = –19, not allowed.

Therefore, this altered remainder 3 is not sufficient.

We still need bigger altered remainder.

24 = 3 × 6 + 6, quotient is 6 and remainder is 6.

Now, the gross dividend = 69,

actual dividend = 69 – 8 × 6 = 21, it is correct.

Write quotient digit 6 and altered remainder 6 at their respective places.

21 ÷ 3 = 7, remainder 0.

$$
\begin{array}{c|cccc|c}
8 & & & & & \\
3 & 6 & 2 & 9 & & 0 \\
& & 3 & 6 & & 6 \\
\hline
& 1 & 6 & 5 & & 20 \\
\end{array}
$$

Gross remainder = 00 = 0,

actual remainder = 0 – 8 × 7 = –56, not allowed.

Therefore, we need altered remainder at the previous step.

$$21 = 3 \times 5 + 6 \qquad \text{(Note it)}$$

So quotient is 5 and altered remainder is 6.

Gross remainder = 60,

actual remainder = 60 – 8 × 5 = 20, it works.

Write quotient digit 5 and remainder 6 at their respective places.

∴   6290 ÷ 38 = 165, remainder 20

Hence, quotient = 165 and remainder = 20.

Example 5. *Divide 37941 by 47.*

Solution. By *Dhvajānka sūtra*, we have :

Procedure :

37 ÷ 4 = 9, remainder 1.

Check: Gross dividend = 19,

actual dividend = 19 – 7 × 9 = –44, not allowed.

Therefore, we need altered remainder.

37 = 4 × 8 + 5, quotient is 8 and remainder is 5.

7					
4	37	9	4	1
	5	3	6	
	8	0	7	12

Write quotient digit 8 and remainder 5 at their respective places.

Gross dividend = 59,

actual dividend = 59 – 7 × 8 = 3.

3 ÷ 4 = 0, remainder = 3.

Write quotient digit 0 and remainder 3 at their respective places.

Next gross dividend = 34,

next actual dividend = 34 – 7 × 0 = 34.

34 ÷ 4 = 8, remainder 2.

Check: Gross remainder = 21,

actual remainder = 21 – 7 × 8 = –45, not allowed.

Therefore, we need altered remainder at previous step.

34 = 4 × 7 + 6, quotient is 7 and remainder is 6.

Now gross remainder = 61,

actual remainder = 61 – 7 × 7 = 12, it works.

Write quotient digit 7 and remainder 6 at their respective places.

∴   37941 ÷ 47 = 807, remainder 12

Hence, quotient = 807 and remainder = 12.

Example 6. *Divide 500001 by 89.*

Solution. By *Dhvajānka sūtra*, we have :

Procedure :

50 ÷ 8 = 6, remainder = 2.

9					
8	50	0	0	0	1
	10	7	8	15	
5	6	1	7	88	

Check : Gross dividend = 20,

actual dividend = 20 – 9 × 6 = –34, not allowed.

Therefore, we need altered remainder at previous step.

$50 = 8 \times 5 + 10$, quotient = 5, remainder = 10.

Write quotient digit 8 and remainder 10 at their respective places.

Next gross dividend = 100,

next actual dividend = $100 - 9 \times 5 = 55$

$55 \div 8 = 6$, remainder 7.

Write quotient digit 6 and remainder 7 at their respective places.

Next gross dividend = 70,

next actual dividend = $70 - 9 \times 6 = 16$

$16 \div 8 = 2$, remainder = 0.

Check : Next gross dividend = 00 = 0,

next actual dividend = $0 - 9 \times 2 = -18$, not allowed.

Therefore, we need altered remainder at previous step.

$16 = 8 \times 1 + 8$, quotient is 1 and remainder is 8.

Write quotient digit 1 and remainder 8 at their respective places.

Next gross dividend = 80,

next actual dividend = $80 - 9 \times 1 = 71$.

$71 \div 8 = 8$, remainder 7.

Check : Gross remainder = 71,

actual remainder = $71 - 9 \times 8 = -1$, not allowed.

Therefore, we need altered remainder at previous step.

$71 = 8 \times 7 + 15$, quotient is 7 and remainder is 15.

Write quotient digit 7 and remainder at their respective places.

Gross remainder = 151,

actual remainder = $151 - 9 \times 3 = 88$.

∴ $500001 \div 89 = 5617$, remainder = 88

Hence, quotient = 5617 and remainder = 88.

Example 7. *Divide 3279421 by 53.*

Solution. By *Dhvajānka sūtra*, we have :

Procedure :

$32 \div 5 = 6$, remainder 2.

Actual dividend = $27 - 3 \times 6 = 9$

$9 \div 5 = 1$, remainder 4.

Next actual dividend = $49 - 3 \times 1 = 46$

$46 \div 5 = 9$, remainder 1.

		3						
5		32	7	9	4	2	1
			2	4	6	5	6
		6	1	8	7	5	46

Check : Next actual remainder = $14 - 3 \times 9 = -13$, not allowed.

Therefore, we need altered remainder at the previous step.

$46 = 5 \times 8 + 6$, quotient 8 and remainder 6.

Next actual dividend = $64 - 3 \times 8 = 40$

$40 \div 5 = 8$, remainder 0.

Check : Next actual dividend = 02 – 3 × 8 = 2 – 24 = – 22, not allowed.

Therefore, we need altered remainder at the previous step.

40 = 5 × 7 + 5, quotient is 7 and remainder is 5.

Next actual dividend = 52 – 3 × 7 = 31.

31 ÷ 5 = 6, remainder 1.

Check : Gross remainder = 12,

actual remainder = 12 – 3 × 6 = – 6, not allowed.

Therefore, we need altered remainder at the previous step.

31 = 5 × 5 + 6, quotient is 5 and remainder is 6.

Gross remainder = 61,

actual remainder = 61 – 3 × 5 = 46.

∴   3279421 ÷ 53 = 61875, remainder 46

Hence, quotient = 61875 and remainder = 46.

! Remark

If the divisor is a two-digit number then the second digit of the divisor is put on top of the flag and the entire division is done only by the first digit and the rest of the work was merely subtraction but if the divisor is a three-digit number then two digits (second and third together) of the divisor are put on top of the flag and division is done only by the first digit of the divisor. Moreover, as two digits of the divisor are put on top of the flag, we allot two places at the right end of the dividend to the remainder portion of the answer and cut off the last two digits of the dividend from the other digits by a vertical (dotted) line. Although *Dhvajānka sūtra* is vertically and crosswise but it does not become apparent till there are atleast 3 digits in the divisor.

If the divisor has atleast three digits, then the division by merely by the first digit (of the divisor) but we have to adopt a different modus operandi on $\overline{U}rdhva$ *Tiryak* lines in respect of the subtraction-portion of the work. However, for the present we have worked only for two-digit divisors.

Exercise 8.4 (S)

Find the quotient and the remainder in the following divisions :

1. 24⌋426

2. 37⌋921

3. 26⌋4307

4. 38⌋6710

5. 86⌋2313

6. 17⌋10000

7. 75⌋37261

8. 57⌋695433

9. 79⌋745607

ANSWERS

EXERCISE 7.1 (S)

1. $\dfrac{5}{8}$, 7, $\dfrac{-3}{-13}$, $\dfrac{-17}{-6}$

2. $\dfrac{-5}{7}$, $\dfrac{4}{-3}$, -6, $\dfrac{-28}{5}$

3. (i) $\dfrac{-4}{10}$, $\dfrac{-6}{15}$, $\dfrac{-8}{20}$, $\dfrac{-10}{25}$

(ii) $\dfrac{6}{-14}$, $\dfrac{9}{-21}$, $\dfrac{12}{-28}$, $\dfrac{15}{-35}$

(iii) $\dfrac{-26}{8}$, $\dfrac{-39}{12}$, $\dfrac{-52}{16}$, $\dfrac{-65}{20}$

(iv) $\dfrac{5}{9}$, $\dfrac{10}{18}$, $\dfrac{15}{27}$, $\dfrac{20}{36}$

4. (i) $\dfrac{-4}{9}$ (ii) $\dfrac{-17}{33}$ (iii) $\dfrac{15}{38}$ (iv) $\dfrac{41}{4}$

5. (i) and (iii)

6. (i) $\dfrac{-3}{2}$ (ii) $\dfrac{-4}{9}$ (iii) $\dfrac{1}{5}$ (iv) $\dfrac{-4}{7}$ (v) $\dfrac{-39}{55}$

EXERCISE 7.2 (S)

1. (i) 0 (ii) $\dfrac{3}{7}$ (iii) $\dfrac{-9}{-5}$ (iv) $\dfrac{-21}{-23}$

2. (i) $-\dfrac{1}{4}$ (ii) $\dfrac{-4}{3}$ (iii) $\dfrac{-3}{4}$ (iv) $-3\dfrac{4}{5}$

3. (i) $<$ (ii) $>$ (iii) $=$ (iv) $<$
(v) $<$ (vi) $>$

4. (i) $-\dfrac{3}{5}$, $\dfrac{-2}{5}$, $\dfrac{-1}{5}$ (ii) $\dfrac{-4}{3}$, $\dfrac{-1}{3}$, $\dfrac{-2}{9}$ (iii) $\dfrac{-3}{2}$, $\dfrac{-3}{4}$, $\dfrac{-3}{7}$ (iv) $\dfrac{-7}{16}$, $\dfrac{5}{-12}$, $\dfrac{9}{-24}$

5. (i) $\dfrac{-7}{15}$, $\dfrac{-11}{20}$, $\dfrac{17}{-30}$ (ii) $\dfrac{2}{-5}$, $\dfrac{19}{-30}$, $\dfrac{-7}{10}$, $\dfrac{-11}{15}$

EXERCISE 7.3 (S)

1. (i) $\dfrac{-2}{11}$ (ii) $\dfrac{-1}{9}$ (iii) $\dfrac{-3}{7}$ (iv) $\dfrac{7}{20}$

2. (i) $\dfrac{-31}{15}$ (ii) $\dfrac{-26}{57}$ (iii) $\dfrac{82}{99}$ (iv) $2\dfrac{4}{15}$

3. (i) $\dfrac{-69}{8}$ (ii) $\dfrac{-31}{72}$ (iii) $\dfrac{-19}{24}$ (iv) $1\dfrac{1}{2}$

4. (i) $\dfrac{10}{13}$ (ii) $\dfrac{-1}{6}$ (iii) $\dfrac{-11}{9}$

5. (i) $\dfrac{23}{63}$ (ii) $\dfrac{1}{195}$ (iii) $-8\dfrac{1}{9}$ (iv) $4\dfrac{23}{24}$

6. $\dfrac{34}{35}$ **7.** $\dfrac{-11}{24}$ **8.** $\dfrac{1}{6}$ **9.** $1\dfrac{14}{19}$

10. (i) $\dfrac{-7}{12}$ (ii) $\dfrac{-30}{49}$ (iii) $\dfrac{10}{9}$ (iv) $\dfrac{-63}{8}$

(v) 1 (vi) $\dfrac{-3}{20}$ (vii) $1\dfrac{1}{2}$ (viii) -10

11. (*i*) −6   (*ii*) $\dfrac{-3}{10}$ (*iii*) $\dfrac{4}{15}$ (*iv*) $\dfrac{-1}{6}$

(*v*) $\dfrac{-14}{13}$ (*vi*) $\dfrac{91}{24}$ (*viii*) $\dfrac{-15}{4}$ (*viii*) $-3\dfrac{5}{33}$

12. $\dfrac{-4}{3}$ **13.** $\dfrac{-9}{7}$

EXERCISE 7.4 (S)

1. (*i*) Terminating   (*ii*) Terminating   (*iii*) Terminating
(*iv*) Non-terminating repeating   (*v*) Terminating   (*vi*) Terminating

2. (*i*) 2·125   (*ii*) 0·00416   (*iii*) 0·0875
(*iv*) has non-terminating repeating decimal expansion   (*v*) 0·0448   (*vi*) 0·158

EXERCISE 8.1 (S)

1. Quotient = 262, remainder = 0   **2.** Quotient = 2390, remainder = 1
3. Quotient = 1472, remainder = 2   **4.** Quotient = 388, remainder = 2
5. Quotient = 1243, remainder = 2   **6.** Quotient = 721, remainder = 2
7. Quotient = 915, remainder = 1   **8.** Quotient = 258, remainder = 8
9. Quotient = 7849, remainder = 1   **10.** Quotient = 8564, remainder = 1
11. Quotient = 378, remainder = 6   **12.** Quotient = 473, remainder = 4

EXERCISE 8.2 (S)

1. Quotient = 21, remainder = 30   **2.** Quotient = 16, remainder = 30
3. Quotient = 223, remainder = 1   **4.** Quotient = 108, remainder = 71
5. Quotient = 61, remainder = 23   **6.** Quotient = 170, remainder = 14
7. Quotient = 1702, remainder = 35   **8.** Quotient = 203, remainder = 69

EXERCISE 8.3 (S)

1. (*i*) $10\big/1, 9\big/4, 8\big/7$ (*ii*) $8\big/6, 7\big/13, 6\big/20$ (*iii*) $9\big/2, 8\big/7, 7\big/12$

2. (*i*) $7\big/4, 6\big/13, 5\big/22, 4\big/31$ (*ii*) $6\big/0, 5\big/8, 4\big/16, 3\big/24$ (*iii*) $6\big/3, 5\big/9, 4\big/15, 3\big/21$

EXERCISE 8.4 (S)

1. Quotient = 17, remainder = 18   **2.** Quotient = 24, remainder = 33
3. Quotient = 165, remainder = 17   **4.** Quotient = 176, remainder = 22
5. Quotient = 26, remainder = 77   **6.** Quotient = 588, remainder = 4
7. Quotient = 496, remainder = 61   **8.** Quotient = 12200, remainder = 33
9. Quotient = 9438, remainder = 5