

● — SUPPLEMENT TO CHAPTER 8 — ●

MULTIPLICATION BY VEDIC METHODS

In VEDIC MATHEMATICS, there are Five-Sūtras for multiplications—*Nikhilam Navataścaramam Daśatah* (Base Method), *Anurūpyena* (Sub-base method), *Ekādhikena* (one more than previous one), *Ūrdhva-Tiryagbhyām* (Vertically and crosswise) and *Shunyant Method* (Reverse Method). Out of these five sutras of multiplication, *Ūrdhva-Tiryagbhyām* is the most general and effective method by which we can find the product of any two natural numbers. It is the crowning gem of all sutras.

Ūrdhva-Tiryagbhyām is a vertically and crosswise method to find the product of two numbers. The number of steps required to get the product is one less than the total number of digits of both the numbers.

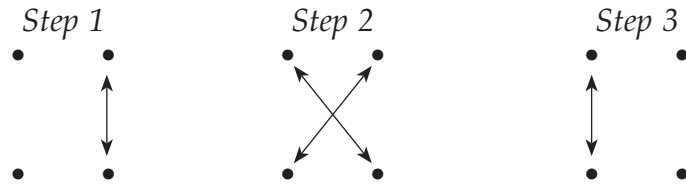
We take some problems to understand the method.

1. Multiplication of two 2-digits numbers

To multiply a 2-digits number by a 2-digits number.

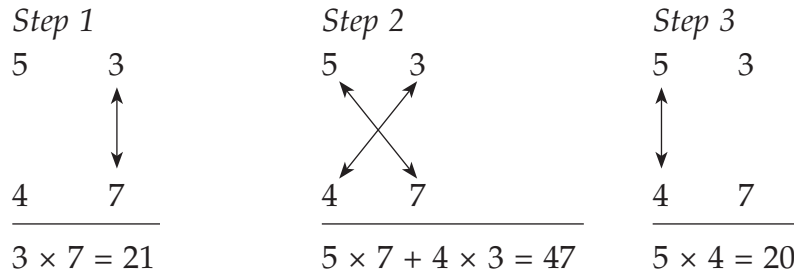
We need 3 steps.

The following diagram may help in remembering the vertically crosswise pattern required for multiplying two 2-digits numbers.



Example 1. Find 53×47

Solution.



Now at each step unit of the result of that step will be the digit of the step and its remaining part is the carry forward and will be added to the next step on its left side.

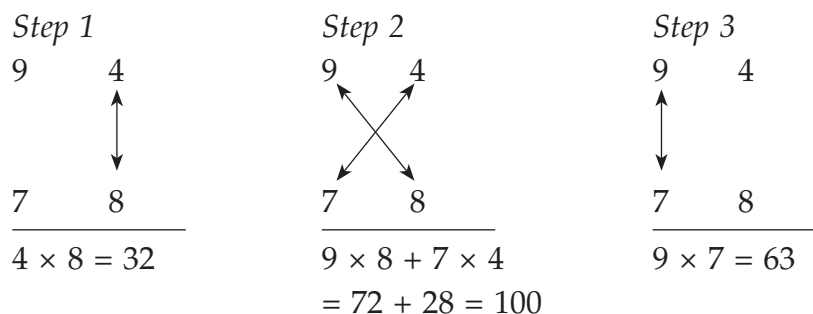
$$\therefore 53 \times 47 = 20/47/21 = 20/4/7/1/2$$

(Carry forward is written at the end of slant line)

$$= 2491$$

Example 2. Find 94×78

Solution.



$$\begin{aligned} \therefore 94 \times 78 &= 63/100/32 = 63/_{10} 0/_{3} 2 \\ &= 7332 \end{aligned}$$

Exercise 8.1 (S)

Multiply the following numbers by using *Ūrdhva-Tiryagbhyām* Method

1. 21×34

2. 37×49

3. 58×77

4. 68×57

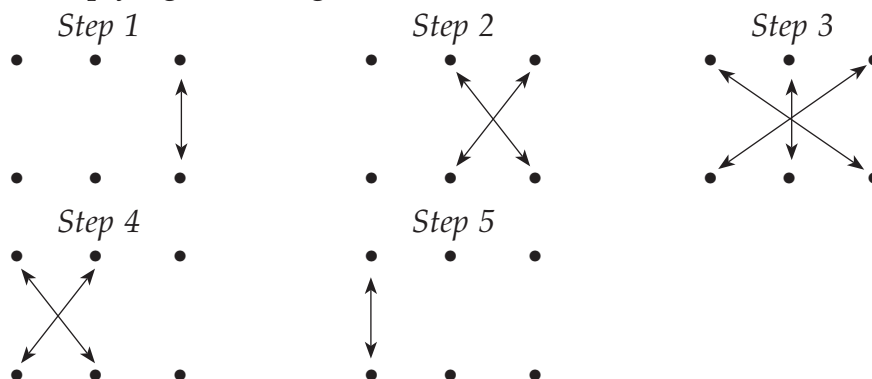
5. 94×78

6. 85×98

2. Multiplication of two 3-digits numbers

To multiply a 3-digits number by a 3-digits number, we need 5 steps.

The following diagram may help in remembering the vertically crosswise pattern required for multiplying two 3-digits numbers.



Example 1. Find 212×543

Solution.

Step 1

$$\begin{array}{ccc} 2 & 1 & 2 \\ 5 & 4 & 3 \\ \hline \end{array}$$

$$2 \times 3 = 6$$

Step 4

$$\begin{array}{ccc} 2 & 1 & 2 \\ 5 & 4 & 3 \\ \hline \end{array}$$

$$\begin{aligned} &2 \times 4 + 5 \times 1 \\ &= 8 + 5 = 13 \end{aligned}$$

Step 2

$$\begin{array}{ccc} 2 & 1 & 2 \\ 5 & 4 & 3 \\ \hline \end{array}$$

$$\begin{aligned} &1 \times 3 + 4 \times 2 \\ &= 3 + 8 = 11 \end{aligned}$$

Step 5

$$\begin{array}{ccc} 2 & 1 & 2 \\ 5 & 4 & 3 \\ \hline \end{array}$$

$$2 \times 5 = 10$$

Step 3

$$\begin{array}{ccc} 2 & 1 & 2 \\ 5 & 4 & 3 \\ \hline \end{array}$$

$$\begin{aligned} &2 \times 3 + 5 \times 2 + 1 \times 4 \\ &= 6 + 10 + 4 = 20 \end{aligned}$$

$$\begin{aligned} \therefore 212 \times 543 &= 10/13/20/11/6 = 10/_{1} 3/_{2} 0/_{1} 1/_{1} 6 \\ &= 115116 \end{aligned}$$

Example 2. Find 739×625

Solution.

Step 1

$$\begin{array}{r} 7 \quad 3 \quad 9 \\ \quad \updownarrow \\ 6 \quad 2 \quad 5 \\ \hline 9 \times 5 = 45 \end{array}$$

Step 2

$$\begin{array}{r} 7 \quad 3 \quad 9 \\ \quad \swarrow \quad \searrow \\ 6 \quad 2 \quad 5 \\ \hline 3 \times 5 + 2 \times 9 \\ = 15 + 18 = 33 \end{array}$$

Step 3

$$\begin{array}{r} 7 \quad 3 \quad 9 \\ \swarrow \quad \updownarrow \quad \searrow \\ 6 \quad 2 \quad 5 \\ \hline 7 \times 5 + 6 \times 9 + 3 \times 2 \\ = 35 + 54 + 6 = 95 \end{array}$$

Step 4

$$\begin{array}{r} 7 \quad 3 \quad 9 \\ \swarrow \quad \searrow \\ 6 \quad 2 \quad 5 \\ \hline 7 \times 2 + 6 \times 3 \\ = 14 + 18 = 32 \end{array}$$

Step 5

$$\begin{array}{r} 7 \quad 3 \quad 9 \\ \quad \quad \quad \updownarrow \\ 6 \quad 2 \quad 5 \\ \hline 7 \times 6 = 42 \end{array}$$

In short, the above 5 steps can be written as :

Step 1 : $9 \times 5 = 45$

Step 2 : $3 \times 5 + 2 \times 9 = 15 + 18 = 33$

Step 3 : $7 \times 5 + 6 \times 9 + 3 \times 2 = 35 + 54 + 6 = 95$

Step 4 : $7 \times 2 + 6 \times 3 = 14 + 18 = 32$

Step 5 : $7 \times 6 = 42$

$$\begin{aligned} \therefore 739 \times 625 &= 42/32/95/33/45 = 42/3 \ 2/9 \ 5/3 \ 3/4 \ 5 \\ &= 42/3 \ 11/875 = 42/4 \ 1/875 \\ &= 461875 \end{aligned}$$

3. Multiplication of a 2-digits number by a 3-digits number

Add a zero in the beginning of 2-digits number to equalise the number of digits and then proceed as multiplication of two 3-digits number.

Example 3. Find 39×574 .

Solution. 39 can be written as 039.

So, we are to find 039×574

Step 1

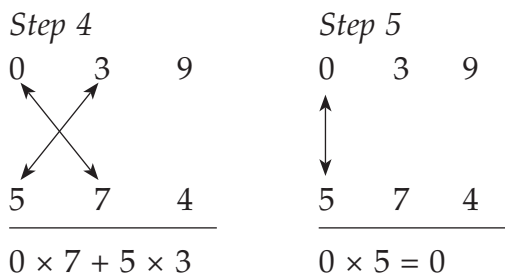
$$\begin{array}{r} 0 \quad 3 \quad 9 \\ \quad \updownarrow \\ 5 \quad 7 \quad 4 \\ \hline 9 \times 4 = 36 \end{array}$$

Step 2

$$\begin{array}{r} 0 \quad 3 \quad 9 \\ \quad \swarrow \quad \searrow \\ 5 \quad 7 \quad 4 \\ \hline 3 \times 4 + 7 \times 9 \\ = 12 + 63 = 75 \end{array}$$

Step 3

$$\begin{array}{r} 0 \quad 3 \quad 9 \\ \swarrow \quad \updownarrow \quad \searrow \\ 5 \quad 7 \quad 4 \\ \hline 0 \times 4 + 5 \times 9 + 3 \times 7 \\ = 0 + 45 + 21 = 66 \end{array}$$



$$\begin{aligned} \therefore 039 \times 574 &= 0/15/66/75/36 = 0/1/5/6/5/6 \\ &= 0/1/5/6/13/86 = 0/1/5/7/3\ 86 \end{aligned}$$

$$\therefore 39 \times 574 = 22386$$

Note

See the multiplication of a 3-digit number by a 2-digit number by using moving multiplier.

Exercise 8.2 (S)

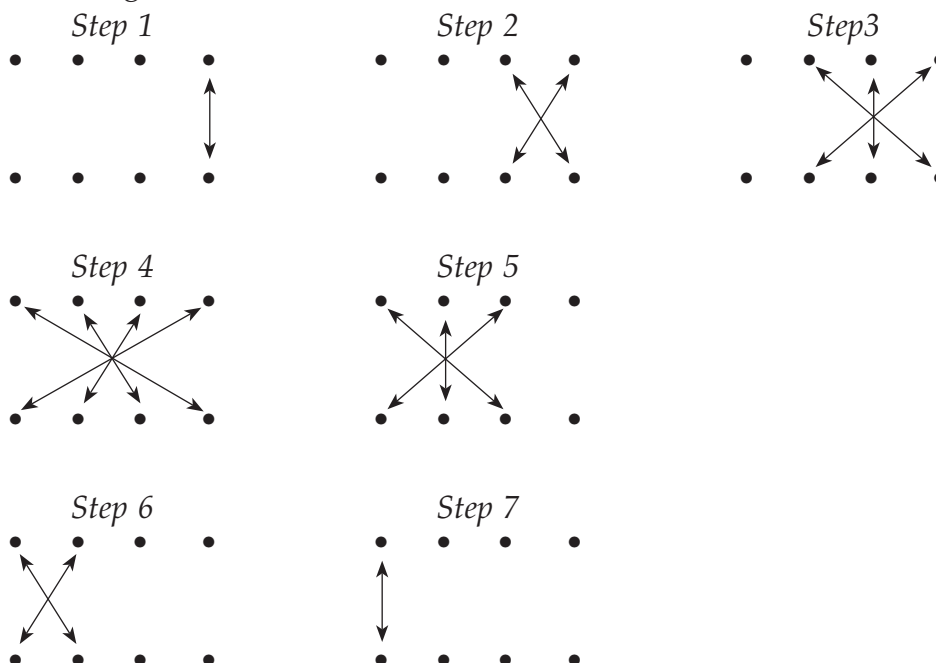
Multiply the following numbers by using *Urdhva-Tiryagbhyām* method :

- | | | |
|---------------------|---------------------|---------------------|
| 1. 231×317 | 2. 412×123 | 3. 646×335 |
| 4. 662×336 | 5. 423×203 | 6. 735×938 |
| 7. 77×475 | 8. 35×374 | 9. 976×43 |

4. Multiplication of two 4-digits numbers

To multiply a 4-digits number by a 4-digits number, we need 7 steps.

The following diagram may help in remembering the vertically crosswise pattern for multiplying two 4-digits numbers :



Example 1. Find 2106×1385

Solution.

Step 1

$$\begin{array}{r} 2 \quad 1 \quad 0 \quad 6 \\ \updownarrow \\ 1 \quad 3 \quad 8 \quad 5 \\ \hline 6 \times 5 = 30 \end{array}$$

Step 2

$$\begin{array}{r} 2 \quad 1 \quad 0 \quad 6 \\ \swarrow \searrow \\ 1 \quad 3 \quad 8 \quad 5 \\ \hline 0 \times 5 + 8 \times 6 \\ = 0 + 48 = 48 \end{array}$$

Step 3

$$\begin{array}{r} 2 \quad 1 \quad 0 \quad 6 \\ \swarrow \updownarrow \searrow \\ 1 \quad 3 \quad 8 \quad 5 \\ \hline 1 \times 5 + 3 \times 6 + 0 \times 8 \\ = 5 + 18 + 0 = 23 \end{array}$$

Step 4

$$\begin{array}{r} 2 \quad 1 \quad 0 \quad 6 \\ \swarrow \updownarrow \searrow \\ 1 \quad 3 \quad 8 \quad 5 \\ \hline 2 \times 5 + 1 \times 6 + 1 \times 8 + 3 \times 0 \\ = 10 + 6 + 8 + 0 = 24 \end{array}$$

Step 5

$$\begin{array}{r} 2 \quad 1 \quad 0 \quad 6 \\ \swarrow \updownarrow \searrow \\ 1 \quad 3 \quad 8 \quad 5 \\ \hline 2 \times 8 + 1 \times 0 + 1 \times 3 \\ = 16 + 0 + 3 = 19 \end{array}$$

Step 6

$$\begin{array}{r} 2 \quad 1 \quad 0 \quad 6 \\ \swarrow \searrow \\ 1 \quad 3 \quad 8 \quad 5 \\ \hline 2 \times 3 + 1 \times 1 = 6 + 1 = 7 \end{array}$$

Step 7

$$\begin{array}{r} 2 \quad 1 \quad 0 \quad 6 \\ \updownarrow \\ 1 \quad 3 \quad 8 \quad 5 \\ \hline 2 \times 1 = 2 \end{array}$$

$$\begin{aligned} \therefore 2106 \times 1385 &= 2/7/19/24/23/48/30 \\ &= 2/7/9/4/3/8/0 \\ &= 2916810 \end{aligned}$$

Example 2. Find 4345×6072

Solution.

$$\begin{array}{r} 4 \quad 3 \quad 4 \quad 5 \\ \times 6 \quad 0 \quad 7 \quad 2 \\ \hline \end{array}$$

In short, 7 working steps can be written as :

Step 1 : $5 \times 2 = 10$

Step 2 : $4 \times 2 + 7 \times 5 = 8 + 35 = 43$

Step 3 : $3 \times 2 + 0 \times 5 + 4 \times 7 = 6 + 0 + 28 = 34$

Step 4 : $4 \times 2 + 6 \times 5 + 3 \times 7 + 0 \times 4 = 8 + 30 + 21 + 0 = 59$

Step 5 : $4 \times 7 + 6 \times 4 + 3 \times 0 = 28 + 24 + 0 = 52$

Step 6 : $4 \times 0 + 6 \times 3 = 0 + 18 = 18$

Step 7 : $4 \times 6 = 24$

$$\begin{aligned} \therefore 4345 \times 6072 &= 24/18/52/59/34/43/10 \\ &= 24/8/2/9/4/3/0 \\ &= 26382840 \end{aligned}$$

Exercise 8.3 (S)

Multiply the following numbers by using *Ūrdhva-Tiryagbhyām* method :

1. 3471×5122 2. 6723×1234 3. 1351×6234 4. 2633×5212

I am sure you have grasped the beauty of this method and understood the simple steps behind this multiplication. You can apply this method no matter how many digits are in each number. After some practice, you will find that the answer can be given in one line and the normal time of calculations can be cut to nearly half.

The rules behind the method are :

Rule 1. Start from the right.

Rule 2. Do a vertical multiplication from right—



Rule 3. Take the next column of digits from the left and do a cross-

multiplication—

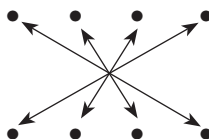


Rule 4. If it a 3-digits \times 3-digits multiplication, rope in the next column of digits and do a cross-multiplication of extremes and vertical multiplication of

middle digits—



Rule 5. If it a 4-digits \times 4-digits multiplication, rope in the next column of digits and do a cross multiplication of extremes and cross-multiplication of the middle digits—



Rule 6. Continue this process till you have reached the extreme left column of digits.

Rule 7. Then start easing off the column of digits on the right and moving leftwards till you reach the extreme column of digits on the left.

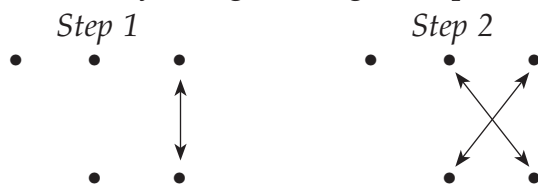
Rule 8. Equalise number of digits in the two numbers by adding zero(s) before the smaller number.

Rule 9. Do not forget to add the carry forwards along the way.

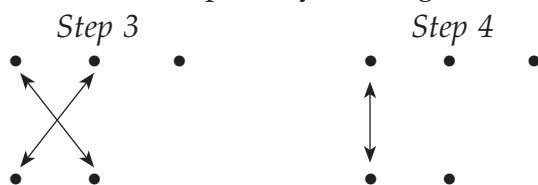
5. Multiplication by using moving multiplier

(1) *Multiplication of a 3-digits number by a 2-digits number by using moving multiplier.*

To multiply a 3-digits number by a 2-digits number, we need 4 steps. The following diagram may help in understanding the vertically crosswise pattern of multiplication by using moving multiplier.



Now move multiplier by one digit to the right.



Example 1. Find 637×54

Solution.

Step 1

$$\begin{array}{ccc} 6 & 3 & 7 \\ & & 5 & 4 \\ \hline \end{array}$$

$7 \times 4 = 28$

Step 2

$$\begin{array}{ccc} 6 & 3 & 7 \\ & & 5 & 4 \\ \hline \end{array}$$

$3 \times 4 + 5 \times 7 = 47$

Move multiplier 54 by one digit to the right

Step 3

$$\begin{array}{ccc} 6 & 3 & 7 \\ & & 5 & 4 \\ \hline \end{array}$$

$6 \times 4 + 5 \times 3 = 39$

Step 4

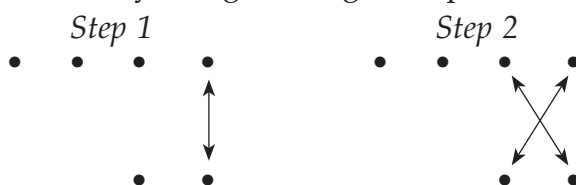
$$\begin{array}{ccc} 6 & 3 & 7 \\ & & 5 & 4 \\ \hline \end{array}$$

$6 \times 5 = 30$

$$\begin{aligned} \therefore 637 \times 54 &= 30/39/47/28 \\ &= 30/9/7/8 \\ &= 34398 \end{aligned}$$

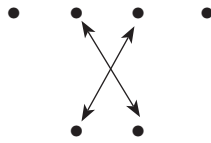
(2) **Multiplication of a 4-digits number by a 2-digits number by using moving multiplier.**

To multiply a 4-digits number by a 2-digits number, we need 5 steps. The following diagram may help in understanding the vertically crosswise pattern of multiplication by using moving multiplier.



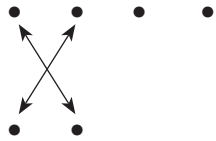
Move the multiplier by one digit to the right.

Step 3



Again move the multiplier by one digit to the right.

Step 4



Step 5



Example 2. Find 3744×67

Solution. Step 1

$$\begin{array}{r} 3 \quad 7 \quad 4 \quad 4 \\ \quad 6 \quad 7 \\ \hline 4 \times 7 = 28 \end{array}$$

Step 2

$$\begin{array}{r} 3 \quad 7 \quad 4 \quad 4 \\ \quad 6 \quad 7 \\ \hline 4 \times 7 + 6 \times 4 = 52 \end{array}$$

Move multiplier 67 by one digit to the right.

Step 3

$$\begin{array}{r} 3 \quad 7 \quad 4 \quad 4 \\ \quad 6 \quad 7 \\ \hline 7 \times 7 + 6 \times 4 = 73 \end{array}$$

Again move the multiplier 67 by one digit to the right.

Step 4

$$\begin{array}{r} 3 \quad 7 \quad 4 \quad 4 \\ \quad 6 \quad 7 \\ \hline 3 \times 7 + 6 \times 7 = 63 \end{array}$$

Step 5

$$\begin{array}{r} 3 \quad 7 \quad 4 \quad 4 \\ \quad 6 \quad 7 \\ \hline 3 \times 6 = 18 \end{array}$$

$$\begin{aligned} \therefore 3744 \times 67 &= 18/63/73/52/28 \\ &= 18/6/3/7/3/5/2/8 \\ &= 250848 \end{aligned}$$

Exercise 8.4 (S)

Find the following products by using *Ūrdhva-Tiryagbhyām* and moving multipliers :

1. 723×46

2. 535×48

3. 687×93

4. 832×33

5. 319×87

6. 3025×48

7. 2617×37

8. 6561×25

9. 3957×93

MULTIPLICATION OF SOME SPECIAL TYPES OF NUMBERS***EKĀDHIKENA SŪTRA*—One more than the previous one.**

This special type of multiplication is for multiplying numbers whose first digits are the same and the last digits add up to 10, 100 etc.

Example 1. Find 75×75

Solution. Step 1 : $5 \times 5 = 25$, which form RHS of answer

Step 2 : $7 \times$ (next consecutive number)

$$= 7 \times 8 = 56, \text{ which form LHS of answer}$$

$$\therefore 75 \times 75 = 5625.$$

Example 2. Find 83×87

Solution. Step 1 : $3 \times 7 = 21$

Step 2 : $8 \times$ (next consecutive number) = $8 \times 9 = 72$

$$\therefore 83 \times 87 = 7221.$$

Example 3. Find 71×79

Solution. Step 1 : $1 \times 9 = 9 = 09$ (make two digits)

Step 2 : $7 \times$ (next consecutive number) = $7 \times 8 = 56$

$$\therefore 71 \times 79 = 5609.$$

Example 4. Find 504×506

Solution. Step 1 : $4 \times 6 = 24$

Step 2 : $50 \times$ (next consecutive integer) = $50 \times 51 = 2550$

$$\therefore 504 \times 506 = 255024.$$

Exercise 8.5 (S)

Find the following products by using *Ekādhikena sūtra*:

1. 45×45

2. 65×65

3. 95×95

4. 52×58

5. 63×67

6. 81×89

7. 202×208

8. 124×126

9. 672×678